A Tilted-Axes Tool for Introductory Mechanics and Mathematics Courses

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An important but challenging problem-solving technique in introductory mechanics is that of using "tilted axes." For inclined plane and centripetal motion problems, using axes that are aligned relative to the ramp direction or the radial direction (rather than the horizontal and vertical) yields equations that are conceptually more straightforward and algebraically more tractable. Of necessity, textbooks tend to present this as a static choice of alternate axes, while experts often think of this as a continuous rotation. Simply printing orthogonal axes onto transparency film produces an inexpensive student handout that bridges this gap in an effective way for those students who have difficulty with the standard textbook presentation.

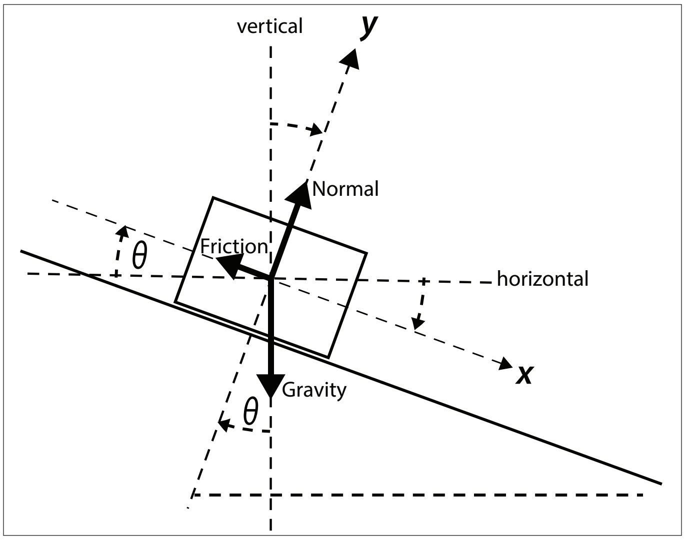

Fig. 1. Block on inclined plane under the influence of three forces. The arc arrows show the rotation of the axes labeled *x* and *y* relative to horizontal and vertical. As students do this rotational manipulation with the transparency, it simultaneously sweeps out both the ramp angle relative to horizontal and the angle between the gravity force and the negative *y*-axis (both labeled *θ*). This demonstrates that those angles are equal with visual immediacy, rather than relying on geometric arguments.

Introduction

One of the more challenging techniques to teach in introductory mechanics is that of using "tilted axes" to simplify the analysis of Newton's laws problems. VanLehn et al.[1] discuss student errors and tutor responses for rotated axes. They measure larger gains in student understanding when (rather than merely implementing the specific case at hand) the tutor presents the general rule of rotating the axes such that one axis is aligned with the vector one seeks.[1] The two principal types of problems that use this technique are ones involving an inclined plane or motion along a curved path.

Newton's second law, $\sum F = ma$, relates quantities that are inherently vectors, but for convenience of calculation, the vector equation is resolved into component equations in perpendicular directions, $\sum F_x = ma_x$ and $\sum F_y = ma_y$. For an inclined plane problem, it is most convenient to use "tilted axes" so that the full acceleration of an object appears in the equation for motion along the direction of the ramp and an acceleration of zero appears in the equation for motion perpendicular to the ramp. If the axis were not "tilted," an acceleration up or down the ramp would require trigonometry to resolve the acceleration into horizontal and vertical components. The reader can appreciate how much more difficult it would be to derive a formula for the acceleration using non-tilted axes; instead of two independent equations each with one unknown quantity for tilted axes, using horizontal and vertical axes would require solving two simultaneous equations each with two unknown quantities. Nunes and Silva present a clear analysis for the full case that also allows tilt of the box relative to the inclined plane.[2]

For motion along a curved path, the choice of appropriate axes is even more fundamental, since the component of acceleration along the path changes the *magnitude* of the particle's velocity, while the component perpendicular to the path changes the *direction* of the particle's velocity.[3] These components are usually expressed as $a_{||} = dv/dt$ and $a_{\perp} = v^2/r$, corresponding to the case of circular motion.

Students typically come into an introductory physics course with extensive practice using axes oriented horizontal and vertical relative to the page or chalkboard, developed from early graphing exercises and courses in geometry and trigonometry. While it is possible to merely tell students to simply "choose" axes with a different orientation, there are pedagogical advantages to have students envision "tilting" the axes in a continuous manner from the conventional to the new orientation. This is a "process" that skilled practitioners often use intuitively, but it is hard to convey in static textbook illustrations. Furthermore, once the idea of smoothly rotating the axes is familiar, a second step of rotating the entire figure allows students to align the new axes with the horizontal and vertical orientation, for which they have acquired skills during trigonometry courses.

Knight solidly documented the difficulty that students have dealing with vectors in general,[4] while Mikula and Heckler evaluated angle geometry and trigonometry errors during inclined plane problems.[5] The tilted-axes tool described here is designed to address these student difficulties and errors.

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Fig. 2. Some students using this tilted axis transparency tool find it to be an easy and enjoyable method of learning to identify the angles in a free-body diagram (compared with geometrical reasoning).

Two prior papers present vector demonstration devices on transparent substrates, but neither are suitable as handouts. Francis presents two large-scale devices designed for classroom use on an overhead projector.[6] One is a single vector that allows persuasive visualization of its components projected onto axes, while the other is a set of orthogonal axes that can be rotated and translated to align with an underlying projection transparency.[6] Wunderlich et al. present a device suitable for visualizing vectors in spherical polar coordinates.[7]

Implementation

In order for each student to experience this process-based technique for themselves, however, a simple handout can be used, consisting of a set of x- and y-axes printed onto clear plastic transparency film. These can be conveniently printed eight or more per letter-size transparency sheet or even onto transparent business cards. (Several templates with different numbers of handouts per sheet are provided as supplementary material online.[8])

For the inclined plane problem, when students use this tool to smoothly rotate the axes, each of the four axis rays ($+x$, $-x$, $+y$, and $-y$) rotate away from horizontal and vertical through angles of equal measure. This demonstrates with immediacy a point in teaching this material that is normally troublesome: that the measure of the angle between the vertically downward weight vector and the negative y-axis is precisely equal to the measure of the angle between the ramp and horizontal, as shown in Fig. 1. For the centripetal acceleration problems, it lets students apply the same tool, framework, and prior practice to this new situation.

Not every student will have a need for such a tool, since many will grasp the concept and the spatial reasoning easily enough. Rather, it is intended to be a significant supplement for those students who need particular help with this concept.

Initial feedback from classroom conversations was that some students in this calculus-based introductory mechanics course particularly liked this tilted-axes tool handout and found it useful enough to want to bring it to exams. (See Fig. 2.) In an extremely preliminary survey after the class at which this material was presented, a surprisingly large number, 94 out of 161 students (58%), chose to use the transparency handout tool to help analyze an inclined plane problem. Of those students who used the tool, 54% felt the tool was helpful, 39% were neutral, and only 6% felt the tool was unhelpful. A formal study is planned in order to measure its impact on student learning and retention.

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