Unsteady Performance of Finite-Span Pitching Propulsors in a Side-by-Side Arrangement

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Experimental measurements are presented on the performance of two finite-span pitching wings with an aspect ratio of AR = 2 interacting in a side-by-side arrangement. Experiments are conducted for various synchronies and cross-stream spacings. The thrust, power and lift coefficients as well as the efficiency are reported for both the individual wings and the collective. The collective thrust and power consumption are found to increase while the efficiency decreases during out-of-phase oscillations ($\phi = \pi$) for close spacings of $Y^* \leq 0.75$. Similarly, the collective thrust and power consumption are found to decrease while the efficiency increases during in-phase oscillations ($\phi = 0$) for spacings of $Y^* \leq 1.125$. In contrast to this trade-off between improved thrust or efficiency, there are conditions such as at $Y^* = 1$ and $\phi \approx \pi/2$ and $3\pi/2$ that give a 20% increase in collective thrust and 17% increase in collective efficiency at the same time. It is further demonstrated that the direction of the collective lift shows a dependence on both the synchrony and the spacing.

Nomenclature

ρ = density $[kg/m^3]$
C_T^* = normalized thrust coefficient
C_P^* = normalized power coefficient
C_L = lift or lateral force coefficient
η^* = normalized propulsive efficiency
Y^* = normalized propulsor spacing in lateral direction
θ_0 = amplitude of motion $[^o]$
ϕ = synchrony or phase difference between the wings
b = thickness of the wing $[m]$
c = chord length $[m]$
f = pitching frequency $[Hz]$
s = span length $[m]$
t = time $[s]$
A = peak-to-peak pitching amplitude of the motion $[m]$
U_∞ = free-stream velocity $[m/s]$
$\overline{P}$ = time-averaged power consumption $[W/s]$
$\overline{T}$ = time-averaged thrust generation $[N/s]$
$\overline{L}$ = time-averaged lift or lateral force generation $[N/s]$
Subscripts
C = collective
1 = $1^{st} wing$
2 = $2^{nd} wing$
iso = isolated wing

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I. Introduction

In nature, many fish propel themselves forward by oscillating their fins in unsteady motions, and quite often they operate in highly organized groups or schools. There are several hypotheses explaining this behavior; ranging from social behaviors, and the need for protection against predators to a reduction in the energetic expenditure for swimming. In any case, a individual in a school will experience fluid dynamic interactions from the other fish in the school. These interactions can potentially change the force production and energetic cost of swimming of an individual and/or the whole collective. Supporting this argument, there have been extensive research efforts explaining energetic benefits of animals in collectives; ranging from a 15% thrust boost for fish schools [2], to $11-14\%$ of metabolic energy savings for white pelicans [3]. So far this problem has been studied mostly for propulsors in in-line arrangements [5–15], as well as mixtures of side-by-side and in-line arrangements [15–17].

The case of foils in side-by-side arrangements has also received some limited attention [1, 15, 17, 18]. In the numerical study of Dong and Lu [18], two wavy foils traveling in side-by-side arrangement were considered and a reduction in the power consumption and an enhancement in the fluid mediated forces was reported for in-phase and out-of-phase cases, respectively. No other synchronies were considered. In the experimental study of Dewey *et al.*[1], a larger synchrony and foil spacing parameter space was investigated, and flow field analysis was provided in connection with force measurements. It was found that in-phase oscillations lead to increases in both thrust generation and power consumption while propulsive efficiency stays at the same level of two foils in isolation whereas for out-of-phase oscillations a decrease in the collective thrust and power and a concurrent increase in collective efficiency occurs.

In a more recent numerical study by Shoele and Zhu [15], two dimensional foils were considered in different canonical arrangements such as in-line, side-by-side, and triangular formation with three foils involved. A performance enhancement was reported for all the arrangements, although the foils in the triangular arrangement were shown to perform poorly compared to other arrangements. Daghooghi and Borazjani [17] conducted one of the first numerical studies investigating the three-dimensional effects on the propulsive performance and flow around self-propelled swimmers in different rectangular formations. Even though they provided information about how the propulsive performance changes with decreasing lateral distance, the streamwise distance between the swimmers was kept constant.

Studies examining the three-dimensional, unsteady interactions in collectives are quite limited [17, 19, 20]. Considering that fish and birds operate under unsteady flow conditions in highly three dimensional formations, it is essential to address this problem by considering the three dimensionality of these interactions. Here, we are interested in understanding the propulsive performance of two finite-span wings interacting in side-by-side arrangements. Force measurements are conducted while the spacing and synchrony between the wings are varied systematically. The results are presented for thrust, lift, and power coefficients as well as the propulsive efficiency for the collective and individual wings in the collective.

II. Experimental Setup and Methods

The experiments have been conducted in a closed loop, free-surface water channel with a test section of 4.9 m length, 0.93 m width, and 0.61 m depth. Throughout all the experiments the free-stream velocity was kept at $U_\infty = 0.093$ m/s, which corresponds to a chord-based Reynolds number of $Re = 11,000$.

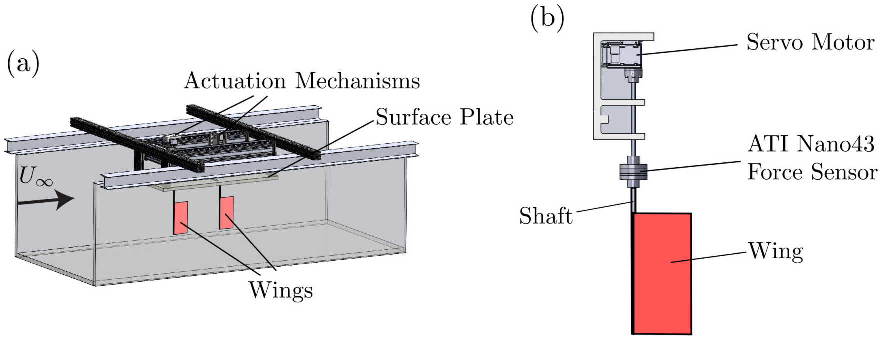

Fig. 1 (a) Experimental setup placed on the top of the water channel. (b) Actuation mechanism used to pitch the wings according to prescribed kinematics.

Two identical wings with a rectangular planform shape (Figure 1), tear-drop cross-section, and a thickness-to-chord

Table 1 Experimental parameters used in the present study.

Parameters		
Y^*	0.625 – 1.25	0.125 increments
ϕ	$0 - 2\pi$	$\pi/12$ increments
k	1	
St	0.25	
θ_0	7.3^o	$(A/c = 0.25)$

ratio of $b/c = 0.07$ are used in the experiments. Each wing has a chord length of $c = 0.095$ m, a span length of $s = 0.19$ m, and an aspect ratio of $AR = 2$. They were 3D-printed with acrylonitrile butadiene styrene (ABS). Each wing is pitched about its leading edge by a Dynamixel MX-64T servo motor with a US Digital E5 optical encoder attached to the shaft tracking the angular position throughout the motion. Wing 1 is pitched with a harmonic motion of $\theta_1(t) = \theta_0 \sin(2\pi f t)$ while wing 2 is pitched similarly as $\theta_2(t) = \theta_0 \sin(2\pi f t + \phi)$, where f is the frequency of the motion, t is the time, θ_0 is the amplitude of motion, and ϕ is the phase difference or *synchrony*. The synchrony between the wings is varied from $0 \le \phi \le 2\pi$ in increments of $\pi/12$ producing 24 phase differentials for each wing arrangement. Throughout this study the time is non-dimensionalized by the period of motion as $t^* = ft$. The non-dimensional cross-stream spacing between the wings, $Y^* = Y/c$, is varied according to Table 1. The frequency of motion and pitching amplitude will be held constant throughout the experiments at $f = 0.98$ Hz and $\theta_0 = 7.3^o$, respectively. This gives a Strouhal number of $St = fA/U_\infty = 0.25$ and a reduced frequency of $k = fc/U_\infty = 1$, where $A = 2c \sin\theta_0$ is the peak-to-peak amplitude of motion.

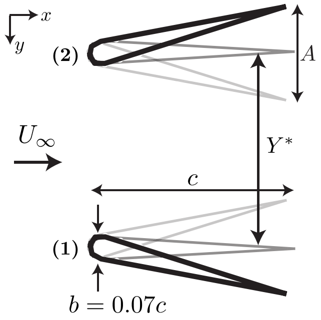

Fig. 2 Schematic of the wings in the side-by-side arrangement. U_∞ is the free stream velocity, Y^* is cross-stream spacing, c is the chord length, b is the thickness of the wing, and A is the peak-to-peak amplitude of the wings.

An ATI Nano43 six-axis force sensor is used to measure the thrust, lift and pitching moments acting on each wing. A second-order central difference scheme is used to calculate the instantaneous angular velocity of each wing from the measured angular position data. The instantaneous total input power is calculated as $P_T(t) = M_\theta \dot{\theta}$. Experiments are conducted in air for an isolated wing and the resultant inertial power data was subtracted from the total power taken in the water tunnel to calculate the power input to the fluid, $P(t)$. The thrust, lift, and power data are time-averaged over 100 oscillation cycles, here presented as mean values of 5 independent trials, and denoted with an overbar such as $\overline{(\cdot)}$. The reported uncertainty is the standard deviation of the five time-averaged values. The coefficient of thrust, C_T, lift, C_L, and power, C_P, and the propulsive efficiency, η, for the wings are defined as,

$$C_T = \frac{\overline{T}}{\frac{1}{2}\rho {U_\infty}^2 cs}, \quad C_L = \frac{\overline{L}}{\frac{1}{2}\rho {U_\infty}^2 cs}, \quad C_P = \frac{\overline{P}}{\frac{1}{2}\rho {U_\infty}^3 cs}, \quad \eta = \frac{C_T}{C_P}, \tag{1}$$

where ρ is the fluid density.

In the current study we also report the collective performance, that is, the combined performance of the wings. The collective performance is important for characterizing the performance of two interacting propulsors on the same animal or device. The collective force and power coefficients as well as the collective efficiency are denoted with a C subscript and they are defined as,

$$C_{T,C} = \frac{\overline{T}_1 + \overline{T}_2}{\rho {U_\infty}^2 cs},\ C_{L,C} = \frac{\overline{L}_1 + \overline{L}_2}{\rho {U_\infty}^2 cs},\ C_{P,C} = \frac{\overline{P}_1 + \overline{P}_2}{\rho {U_\infty}^3 cs},\ \eta_C = \frac{C_{T,C}}{C_{P,C}}. \tag{2}$$

Note that the collective force and power coefficients use the combined wing planform area, that is $2cs$, cancelling the one-half in the denominator of coefficient definitions. The force and power coefficients and the efficiency will be reported as normalized values that are compared to their equivalent isolated wing values. The normalized coefficients and efficiency are,

$$C_T^* = \frac{C_T}{C_{T,iso}}, \qquad C_P^* = \frac{C_P}{C_{P,iso}}, \qquad \eta^* = \frac{\eta}{\eta_{iso}}. \tag{3}$$

Here the single wing metrics are compared to the values of a single isolated wing while the collective wing metrics are compared to the combined values of two isolated wings.

III. Results

A. Propulsive Performance of an Isolated Wing

The forces acting on an isolated wing were measured to determine the baseline for this study. As it was described in Section II, performance coefficients measured for the interaction cases are normalized with the isolated wing values reported in Table 2. The measurements were made over 100 oscillation cycles and for 10 different trials. The lift coefficient value for the isolated wing is not given here, since lift generation for this case is zero in time-average. In relation to this, lift coefficients measured from the interaction cases are shown as their real values.

Table 2 Propulsive performance of an isolated wing at $St = 0.25$, $k = 1$ and $Re = 11,000$.

$C_{T,iso}$	0.20 ± 0.01
$C_{P,iso}$	0.76 ± 0.001
η_{iso}	0.26 ± 0.02

B. Propulsive Performance for Wings in a Side-by-Side Arrangement

Force measurements are conducted for both wings, however, since there is a symmetry to the problem, only the performance metrics of wing 1 are shown here. The thrust, and power coefficients, and propulsive efficiency of wing 2 is a mirror of that of wing 1 about $\phi = \pi$. In addition, lift is mirrored around $\phi = \pi$ for wing 2 and it is multiplied by minus one to properly account for the symmetry of the problem.

Figure 3 presents the normalized thrust, power and propulsive efficiency, along with the lift coefficient of wing 1 as a function of both the spacing and synchrony. Depending upon the synchrony and spacing there can be increases or decreases in thrust, power, and efficiency compared to a wing in isolation. A peak in thrust can be observed near $\phi = \pi/2$ while a trough occurs near $\phi = 3\pi/2$. The peaks and troughs in power are somewhat offset from those of thrust at $\phi = \pi$ and $\phi = 0$, respectively. The variation in the thrust and power from an isolated wing decay as the spacing is increased. In contrast, the variations in the efficiency show a slower decay as the spacing is increased. In fact, the peak efficiency increase over a wing in isolation is around 40% at $\phi \approx \pi/2$ and it extends over the spacings of $0.625 \leq Y^* \leq 1.25$. For the individual performance the increase in thrust and efficiency are nearly coincident, which is in contrast to previous two-dimensional studies [1].

For in-phase oscillations ($\phi = 0$ and 2π), the wing is exposed to lateral forces in $+y$ direction (Figure 2), which indicates that the wing is being pushed away from the other wing. On the other hand, for out-of-phase ($\phi = \pi$) oscillations, peak lateral forces are in the $-y$ direction, which indicates that the the is being pulled towards the other wing. This trend is more accentuated for the lower spacings, $Y^* \leq 0.875$, while at spacings $Y^* \geq 1.125$ the lift generation decays to nearly zero for all of the synchronies even though thrust and efficiency improvements can be observed.

Figure 4 presents the normalized thrust, power, efficiency, and lift of the collective of two interacting wings as a function of synchrony and cross-stream spacing. The thrust contour shows a thrust improvement within the synchrony

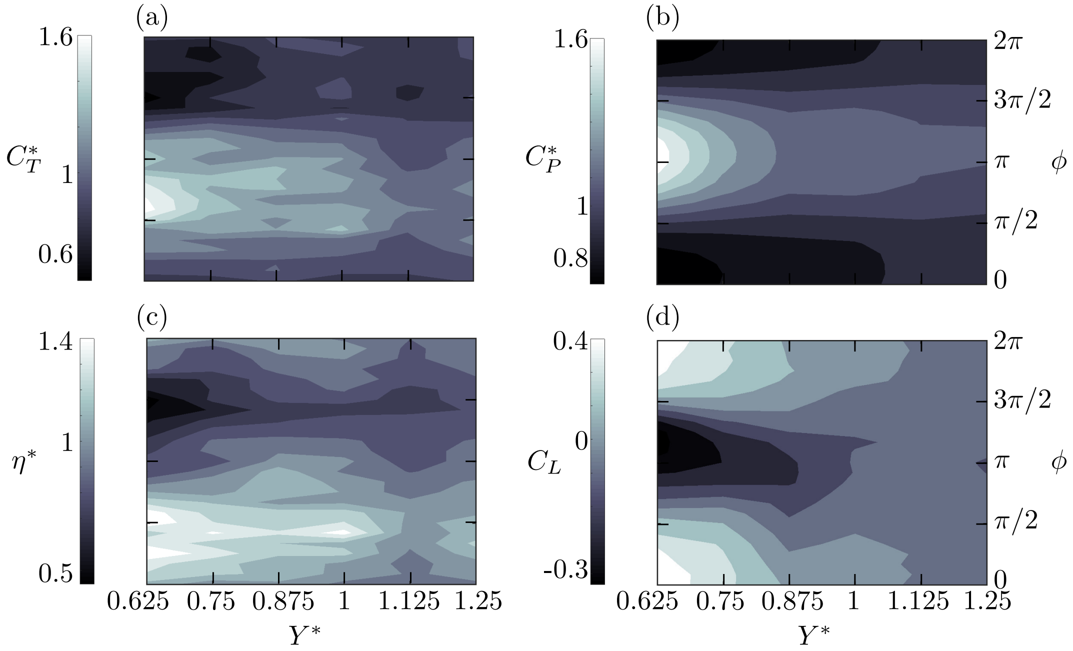

Fig. 3 Wing 1 performance as a function of synchrony and spacing: (a) Normalized thrust, (b) normalized power, (c) normalized efficiency, and (d) lift coefficient.

range of $\pi/2 \leq \phi \leq 3\pi/2$ for the spacings of $Y^* < 1.125$. Just beyond this spacing, the collective under-performs in terms of both thrust generation and propulsive efficiency compared to two wings in isolation. Similar to the wing 1 (Figure 3), power consumption for the collective becomes reduced to the level of two wings in isolation as the wings are moved further apart from each other in the cross-stream direction. Previously, in the study of Dewey *et al.*[1], thrust, and power coefficients and propulsive efficiency were reported for a collective of two foils (two-dimensional flow) in side-by-side arrangements for $Y^* = 0.5$. In the present study, the spacing could not be reduced to that spacing due to the restriction of the wings touching each other during the oscillation cycle. If the closest spacing data is compared with Dewey *et al.*[1], it can be deduced that peak thrust and power show much higher values for the collective of foils than of wings, although the timing of thrust generation, and power consumption show similar trends with synchrony between collective of the foils and the wings. The efficiency also shows similar trends with the study of Dewey *et al.*[1] for the closest spacing of $Y^* = 0.625$, where peak collective efficiency occurs near a synchrony of $\phi = 0$ while the minimum collective efficiency occurs a $\phi = \pi$. This leads to a trade-off between increased efficiency and increased thrust at the closest spacing as the synchrony is varied. In contrast to this trade-off and previous two-dimensional studies [1], here, the at collective data reveals that at synchronies and a cross-stream spacing of $\phi = \pi/2$, $3\pi/2$ and $Y^* = 1$, respectively, the collective efficiency increases by 15% and the collective thrust is increases by 20% at the same time. The concurrent rise in thrust and efficiency is driven by an improvement in thrust while the power remains at the levels of two isolated wings.

As expected, the lift generation from the collective has peak values for the closest spacings and they vary from positive to negative values as the synchrony is varied. What is surprising is that as the spacing is increased from its smallest value the lift switches sign twice. Without further flow measurements it is unclear why this happens. Additionally, at the locations of a concurrent rise in thrust and efficiency circled in red in Figure 4, there is nearly zero collective lift ($|C_L| < 0.03$).

IV. Conclusion

The propulsive performance of finite-span pitching wings operating in a side-by-side arrangements were found to depend on the synchrony as well as cross-stream spacing for both the collective and the individuals operating in the collective. For an individual wing, there was a concurrent peak in thrust and efficiency near the synchrony of $\phi = \pi/2$ and a concurrent trough in thrust and efficiency near $\phi = 3\pi/2$. In contrast to previous studies [1], the peaks and troughs

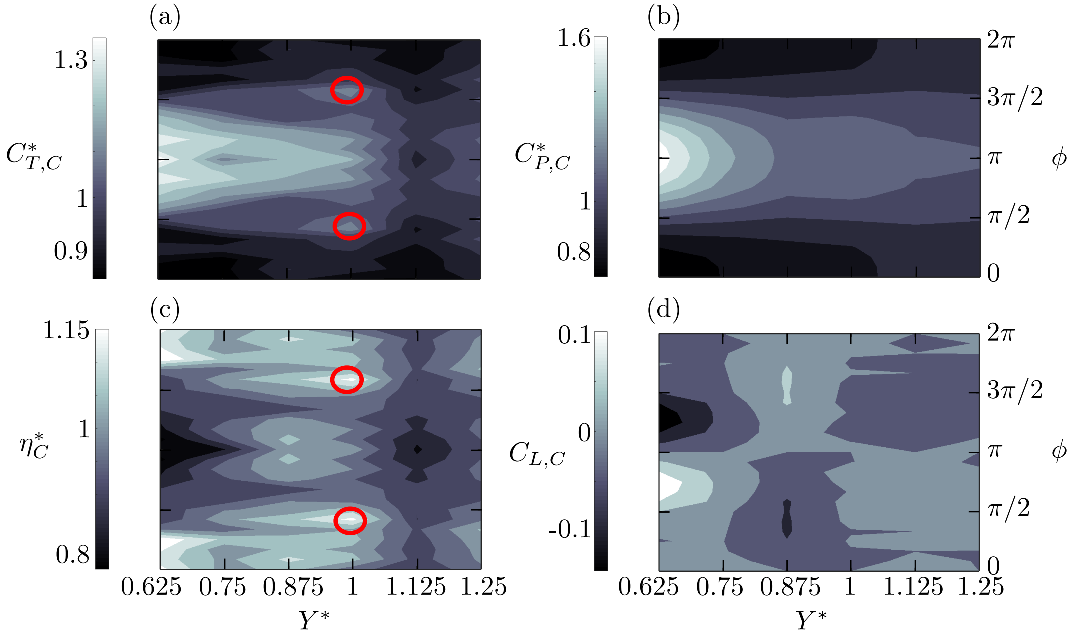

Fig. 4 Collective performance as a function of synchrony and spacing: (a) Normalized thrust, (b) normalized power, (c) normalized efficiency, and (d) lift coefficient. Red circles are marking a condition of concurrent thrust and efficiency improvement compared to two wings in isolation.

in power were not coincident with peaks and troughs in thrust and they occurred near $\phi = \pi$ and $\phi = 0$, respectively. Additionally, it was discovered that at a cross-stream spacing of $Y^* = 1$ and synchronies of $\phi \approx \pi/2$ and $3\pi/2$, there was a concurrent rise in the collective thrust and efficiency of 20% and 17%, respectively, with negligible collective lift generation. Finally, the direction of collective lift generation shows a surprising dependence on the spacing where it switches sign as the spacing is increased. Future work will focus on corresponding flow measurements in order to gain a more complete understanding of the findings revealed in this study.

Acknowledgments

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