

# The swimming of manta rays

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**Abstract.** Aquatic animals propel themselves using a wide variety of mechanisms. In manta rays, propulsion is achieved by combining oscillating and undulating motions of flexible surfaces. We describe two principal sets of experiments to study the effects of excitation and flexibility on the wake flowfield: undulating and flapping three-dimensional fins of elliptical planform, and pitching two-dimensional panels of rectangular planform with varying flexibility. To interpret the results on thrust and efficiency, we propose scalings for aspect ratio and flexibility, and develop a stability analysis called wake resonance theory. Here we focus on the insights provided by wake resonance theory.

**Keywords:** Keywords are separated by half-angle origin.

## 1 Introduction

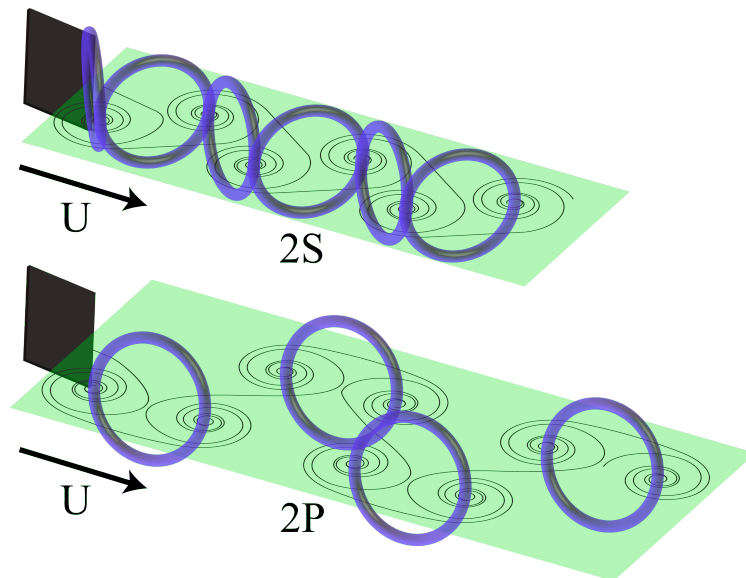
Aquatic animals propel themselves using a wide variety of mechanisms. Jellyfish, squid and salps use pulsatile jets to produce locomotion. Whales, dolphins and many fish use body and caudal fin oscillations, while penguins, sea turtles and some fish employ paired pectoral fin locomotion to propel themselves forward. Sfakiotakis et al. (1999) has classified fish propulsion mechanisms based on the types of fins and the kinematic motions that are employed. For cruise locomotion, where fish are oscillating their fins in a periodic motion, there are two main categories for all fish species; body and/or caudal fin (BCF) locomotion and median and/or paired fin (MPF) locomotion. BCF swimmers propagate a traveling wave down the length of their bodies. They are classified along an undulation continuum where the wavelength of motion ( $\lambda$ ) as compared to the body length ( $L$ ) ranges from values less than or near one, such as in eels using *anguilliform* motion, to values that are much greater than one, such as in tuna using *thunniform* motion. The more undulatory *anguilliform* motion typically engages nearly the entire body in active swimming whereas the less undulatory *carangiform* or *thunniform* motions only engage part of the body or the caudal fin alone.

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MPF fish include boxfish, seaperch, sunfish, and notably rays. Similar to BCF swimmers, rays, which include but are not limited to stingrays, cownose rays, eagle rays and manta rays, have been classified based on the nondimensional wavelength of the traveling wave that is propagated down their pectoral fins (Rosenberger, 2001). Benthic rays such as the Atlantic stingray typically utilize highly undulatory motions ( $\lambda/L < 1$ ) whereas pelagic species such as eagle and manta rays use less undulatory motions ( $\lambda/L > 2$ ).

Regardless of the swimming mechanism employed, all of the oscillating motions used by aquatic species shed a series of vortices into their wakes. These vortex wake structures impart momentum and impulse to the fluid environment causing a equal and opposite force to propel the animal forward. In the case of pulsatile jet production, individual vortex rings are ejected in the downstream direction, which can be seen to produce a jet structure through the center of the rings (Dabiri et al., 2005). For BCF and MPF swimmers, typically a series of connected vortex rings or pairs of laterally ejected vortex rings are shed into the wake (Lauder and Fish, 2006). When a partial image velocimetry (PIV) in a plane at the midspan of a fin is used to measure the velocity field in the wake, then a two-dimensional slice of the three-dimensional vortex rings may be investigated (see Figure 1.1). Here, the series of connected vortex rings is seen to be a three-dimensional analog to a reverse von Kármán street also known as a 2S wake struc-



**Fig. 1.1** Typical three-dimensional vortex wake structures produced by swimming animals. The top is a reverse von Kármán street otherwise known as a 2S wake, where two single vortices are shed per cycle. The bottom is a 2P wake where two pairs of vortices are shed per cycle.

ture where two single vortices are shed per oscillation cycle. The pairs of laterally ejected vortex rings are seen to be the three-dimensional analog to what is known as a 2P wake structure where two pairs of vortices are shed per flapping cycle. In flapping airfoils that model animal locomotion even more exotic wake structures have also been observed (Lentink, 2008).

In nature there seem to be two ubiquitous design principles for swimming locomotion at high Reynolds numbers; animals utilize unsteady fluid mechanics and flexible appendages to swim effectively. This is quite a contrast to our design principles for current underwater vehicle technology that employ steady fluid mechanics principles and rigid propulsors. Exploring and understanding the swimming mechanisms of aquatic species may help to develop novel underwater vehicle designs that could outperform current technology.

Rays offer a unique biological focal point to explore MPF propulsion since different ray species span the undulation continuum. Specifically in manta rays, propulsion is achieved by combining oscillating and undulating motions of flexible surfaces. Oscillations produce a spanwise flapping, and undulations produce a traveling wave that moves in the downstream direction (see Figure 1.2). We are interested in studying the unsteady hydrodynamics of such motions to understand and model the wake structure. Ultimately, our goal is to understand the connection between the wake structure and the swimming performance exhibited by manta rays. Experiments suggest a rich set of phenomena exist, depending on the non-dimensional frequency of flapping, the wavelength of the undulation, the aspect ratio of the fin, and its bending rigidity. Under certain conditions, simple 2S wake structures are observed that bear a strong resemblance to the structure of co-flowing jets and wakes. In other cases, 2P bifurcating wakes are seen, and both cases appear to correspond to a peak in efficiency.

We describe two principal sets of experiments to study the effects of excitation and flexibility on the wake flowfield: undulating and flapping three-



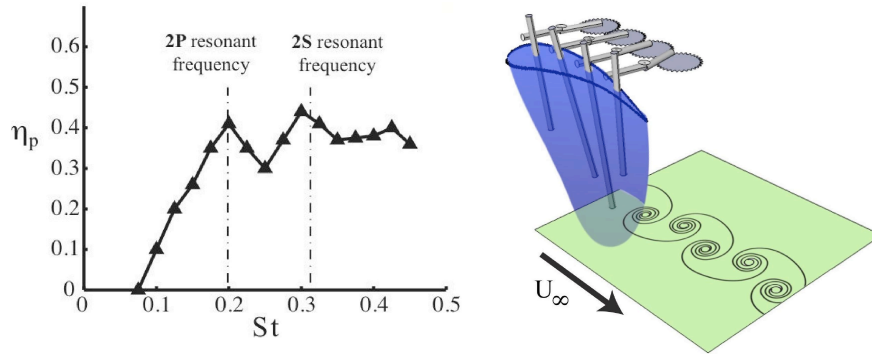
**Fig. 1.2** Manta birostris, off the coast of Yap. Photo credit: Keith Moored.

dimensional fins of elliptical planform, and pitching two-dimensional panels of

rectangular planform with varying flexibility. To interpret the results on thrust and efficiency, we propose scalings for aspect ratio and flexibility, and develop a stability analysis called wake resonance theory. Here we focus on the insights provided by wake resonance theory.

## 2 Undulating and Flapping Three-Dimensional Elliptical Fin

The motivating result is shown in Figure 2.1, which displays the efficiency  $\eta_p$  of a mechanical analog of a manta ray pectoral fin, actuated using four rigid spars to produce an undulatory motion, as implemented by Clark & Smits (2006). Two observations are important: the maximum efficiency exceeds 40%, and there are two peaks in efficiency, one at a Strouhal number  $St = 0.2$  corresponding to a 2P wake and one at a Strouhal number of 0.3 corresponding to a 2S wake. Here,  $St = fA/U$ , where  $f$  is the frequency of actuation,  $A$  is the peak-to-peak amplitude of the trailing edge motion at the half-span, and  $U$  is the freestream velocity.

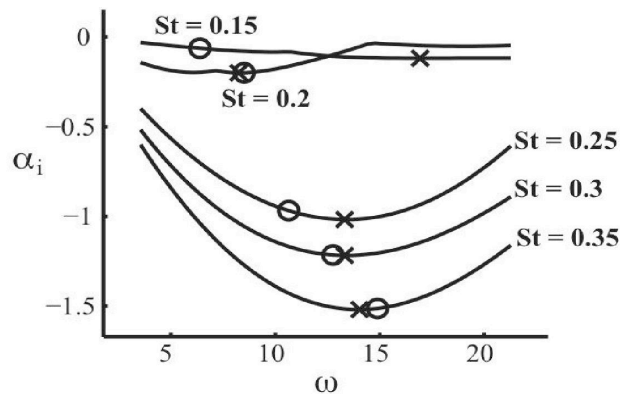


**Fig. 2.1** Left: Efficiency of a batoid-like fin actuated in undulatory motion with a wavelength four times the chord length (adapted from Clark & Smits, 2006). Right: Fin schematic, showing 2S wake.

To provide insight into this behavior, we can examine the stability of the time-averaged wake profile. A linear spatial stability analysis is used to find the frequency of maximum spatial growth, that is, the hydrodynamic resonant frequency of the time-averaged jet. The details are given by Moored et al. (2012a). It is found that: (i) optima in propulsive efficiency occur when the driving frequency of a flapping fin matches the resonant frequency of the jet profile; (ii) there can be multiple wake resonant frequencies and modes corresponding to multiple peaks in efficiency; and (iii) some wake structures transition from one pattern to another when the wake instability mode transitions.



The results are illustrated in Figure 2.2, which shows stability curves formed from the most unstable eigenvalues at a given forcing frequency. There are two distinguished frequencies marked on each stability curve. The driving frequencies at which the velocity profiles and the stability curves were generated are marked with circles. The most unstable frequency of the velocity profile is marked with a cross. When the driving frequency is coincident with the most unstable frequency of a velocity profile, we define this special frequency as a wake resonant frequency. It is postulated that when the fin is operated at a wake resonant frequency then the spatial growth rate of instability waves will be maximized and amplify the mean flow per unit input energy leading to a peak in efficiency (Triantafyllou et al., 1993). The dashed lines in Figure 1 correspond to the two cases shown in Figure 2 where wake resonant frequencies are found, and indeed there is a close correspondence. The theoretical framework is termed wake resonance theory, which is a compilation of conclusions drawn by Moored et al. (2012, 2013). The analysis, although one-dimensional, captures the performance exhibited by a three-dimensional propulsor as seen in this case of an elliptical fin, showing the robustness and broad applicability of the technique.



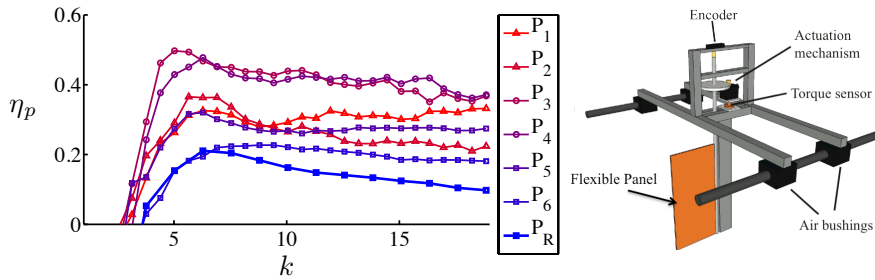
**Fig. 2.2** Stability curves for five velocity profiles taken from PIV data measured by Dewey et al. (2011). The x mark the most unstable frequency of the stability curve while the o mark the driving frequency used to generate the velocity profile (adapted from Moored et al., 2012).

### 3 Pitching Two-Dimensional Flexible Panels

Although rigid materials can be found throughout nature such as bone and nacre (Jackson et al., 1988), manta rays and other aquatic species flap and undulate fins that are made from flexible materials (Lauder & Madden, 2007). We propose that flexible materials have been evolutionarily selected for propulsors, to gain a hydrodynamic advantage over propulsors that use rigid materials. To explore this hypothesis, Dewey et al. (2012) isolated the role of flexibility in unsteady locomotion by experimentally measuring the performance of a pitching panel apparatus

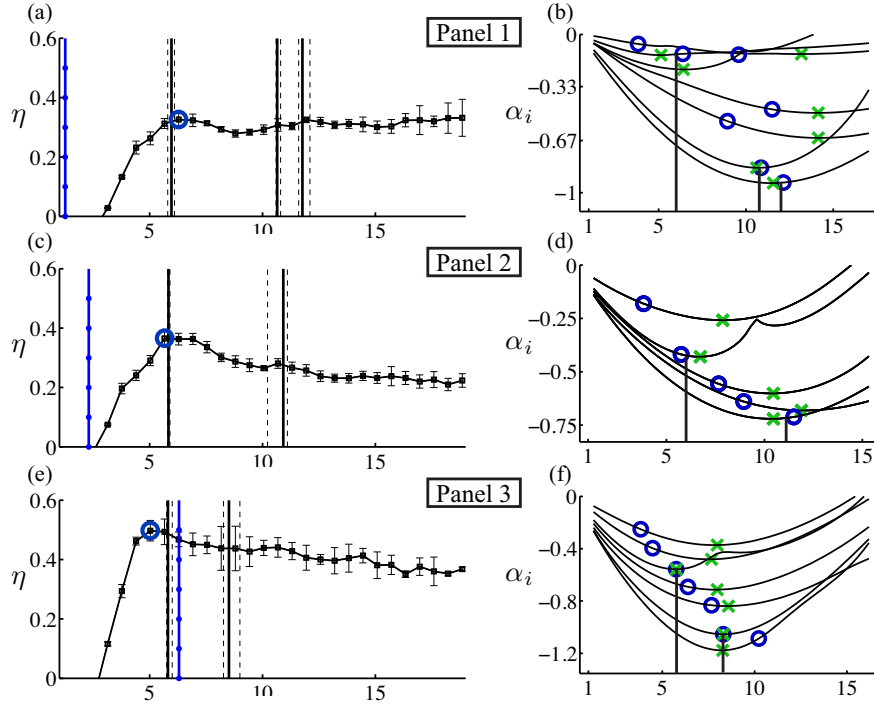
(see Figure 3.1) that produced a two-dimensional flowfield and where numerous panels of varying flexibility were used.

The swimming performance of panels with varying flexibility was measured through the use of the coefficient of thrust,  $C_T = T/(1/2 \rho S U^2)$ , the coefficient of power,  $C_P = P/(1/2 \rho S U^3)$ , and the Froude efficiency,  $\eta_p = C_T/C_P$ . The time averaged thrust,  $T$ , the time averaged power input to the fluid,  $P$ , the fluid density,  $\rho$ , the planform area,  $S$ , and the free-stream flow speed,  $U$ , are all used in the nondimensional force and power coefficients. Dewey et al. (2012) observed that thrust production could be more than doubled for a flexible panel as compared to a rigid panel at the same operating frequency. Beyond the thrust production, the power consumed by a flexible panel can be reduced as compared to a rigid panel with the net effect being that flexible panels can be more than twice as efficient as their rigid counterparts. The Froude efficiency for seven panels is shown in Figure 3.1a. Panel  $P_1$  is the most flexible panel and the rigidity increases to panel  $P_6$ , which is the least flexible panel. Panel  $P_R$  is a rigid panel. The efficiency data from Dewey et al. (2012) shows that there is an optimal flexibility (panels  $P_3$  and  $P_4$ ) where the efficiency is maximized across a range of reduced frequencies. The reduced frequency,  $k = 2\pi fc/U$ , where  $f$  is the driving frequency and  $c$  is the chord length.



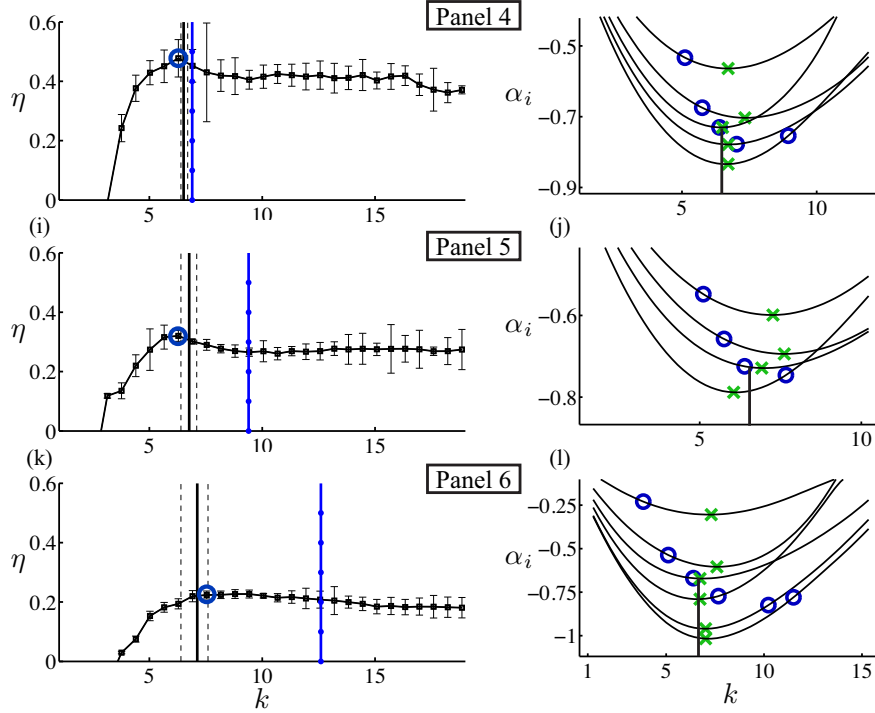
**Fig. 3.1** (a) Froude efficiency of seven panels of varying flexibility. The panels range from the most flexible,  $P_1$ , to the least flexible,  $P_6$ , with the rigid panel denoted as  $P_R$ . (b) Pitching panel apparatus (adapted from Dewey et al., 2012).

Previously, it was shown that local peaks in efficiency occurred when the driving frequency of a fin was matched to its wake resonant frequency. The principles of wake resonance theory have been extended to flexible propulsors, specifically to understand the observed phenomena of an optimally efficient flexibility. Moored et al. (2012b) hypothesized that driving a *flexible* fin at a wake resonant frequency would also lead to local peaks in efficiency and that operating at a structural resonant frequency was not sufficient to achieve a local peak in efficiency. In fact, the optimally efficient flexibility was proposed to occur when the structural resonant frequency of a flexible panel was nearly aligned with a wake resonant frequency.



**Fig. 3.2** Panels one through three: (left) Efficiency curves marked with the wake resonant frequencies (solid black lines) and the structural resonant frequencies (dot-solid blue lines). The dashed black lines denote wake resonant frequency uncertainty. (right) Stability curves generated from the linear stability analysis of velocity profiles extracted from the time averaged PIV data (adapted from Moored et al., 2012b).

To show that the optimal flexibility did indeed occur under these conditions, the same linear stability analysis was applied to the flowfields generated by flexible fins of rectangular planform. The wake resonant frequencies and structural resonant frequencies of the flexible panels are marked on the efficiency data for each panel in Figures 3.2 and 3.3. The associated stability curves generated from the linear stability analysis of the PIV data are also shown. Here, as hypothesized the efficiency peaks in a global sense when the structural resonant frequencies of the panels are nearly aligned with their wake resonant frequencies, which is seen for panels  $P_3$  and  $P_4$ . It can also be seen that operating at a structural resonant frequency does not lead to local peak in efficiency as suggested in Moored et al. (2012b).



**Fig. 3.3** Panels four through six: (left) Efficiency curves marked with the wake resonant frequencies (solid black lines) and the structural resonant frequencies (dot-solid blue lines). The dashed black lines denote wake resonant frequency uncertainty. (right) Stability curves generated from the linear stability analysis of velocity profiles extracted from the time averaged PIV data (adapted from Moored et al., 2012b).

## 4 Conclusions and Future Directions

We have reviewed recent literature on wake resonance theory, which is a set of conclusions that clarify the mechanisms leading to efficient unsteady locomotion at high Reynolds numbers. Wake resonance theory states that local peaks in efficiency are achieved when the driving frequency of motion is matched to a wake resonant frequency for either flexible or rigid propulsors. The theory also states that there may be multiple wake resonant frequencies leading to multiple peaks in efficiency. The case of multiple wake resonant frequencies was first clearly shown for an undulating elliptical fin. In this case, the first peak in efficiency was

shown to relate to a 2P wake structure while the second efficiency peak was paired with a 2S wake structure. This observation established that both 2S *and* 2P wake structures can lead to efficient locomotion and that both wake structures had independent wake resonant frequencies. Many caudal fin swimmers such as tuna typically produce a 2S wake structure while other elongated fish such as eels typically produce a 2P wake structure (Tytell et al., 2010, Fish & Lauder, 2006). Now, it may be concluded that both wakes may be efficient ways for animals to control the fluid flow around their bodies and appendages. The undulatory motion exhibited by manta rays has been shown to produce both wake structures depending on the nondimensional wavelength of the motion. Lastly, recent literature has shown that there is a flexibility that optimizes the efficiency of swimming. A tuning condition has been shown to characterize the optimal flexibility. When a structural resonant frequency is tuned to a wake resonant frequency then the global optimal efficiency of a flexible propulsor is achieved.

Thus far, wake resonance analysis has only been applied to fixed velocity bio-robotic devices. The next step would be to explore whether wake resonance theory applies to free-swimming devices and computations, which is a step closer to autonomous underwater vehicles and animal swimming. We believe that tuning to a wake resonant frequency is a ubiquitous design principle used throughout nature, however this has not been directly demonstrated. To clearly show that animals do indeed tune into the resonances of their wakes, PIV data from animals must be analyzed. If animals do operate at their wake resonant frequencies then a stronger statement than previous literature may be concluded that animals are tuned for high efficiency. Currently, it is proposed that at a wake resonant frequency the time-averaged jet produced behind an unsteady propulsor is amplified per unit input energy leading to the observed local peaks in efficiency. A detailed energy budget analysis of the mean flow production must be explored to confirm the amplification-efficiency mechanism. Lastly, the Orr-Sommerfeld equations may be studied to help determine scaling laws of Froude efficiency for unsteady swimmers and flyers.

We have made considerable progress in understanding the swimming of manta rays. Not only have we determined underlying principles leading to their efficient locomotion, but we have also gained understanding into their fin kinematics (Fish et al., 2011), swimming performance (Clark and Smits, 2006, Moored et al., 2011b), maneuverability (Parson et al., 2011), biomechanics (Russo et al., 2011) and wake structure (Dewey et al., 2011). Scaling laws (Dewey et al. 2012) and effective artificial fins (Moored et al. 2011a) have also been developed to aid in the engineering of ray-like mechanisms. This new understanding of manta rays, we believe, will lead to a more complete understanding of the aerodynamics and hydrodynamics of flying and swimming animals, and to the development of a new class of bio-robotic underwater vehicles that are tuned for high efficiency and may outperform their classic counterparts.

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