ACC 2023 Workshop

Principles of Risk Quantification in Networked Control Systems

Guangyi Liu and Nader Motee

Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

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Principles of Risk Quantification in Networked Control Systems

Introduction to Risk Measures

Guangyi Liu

Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

8:35 am - 9:15 am

Section Overview

- Probability Theory
- Risk Measures
- Coherent Risk Measures
- Distributionally Robust Risk Measures

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Probability Theory: Probability Measure and Sigma Algebra

Goal: To quantify notions of randomness and chances formally.

Probability Measure: For a given sample space Ω, and subsets $A, B \subset \Omega$, we want

- $\mathbb{P}(\Omega) = 1$, and $\mathbb{P}(\emptyset) = 0$.
- $\mathbb{P}(\mathrm{A}) \in [0,1]$
- $\mathbb{P}(A \cup \mathrm{B}) = \mathbb{P}(\mathrm{A}) + \mathbb{P}(\mathrm{B})$ if A, B are disjoint
- $\mathbb{P}(\cup_{j=1}^{\infty} A_j) = \sum_{j=1}^{\infty} \mathbb{P}(\mathrm{A}_j)$ if A_i, A_j are pairwise disjoint

Sigma Algebra: Let Ω be a set. A collection of subsets $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ is called a sigma algebra if:

- $\emptyset, \Omega \in \mathcal{A}$
- $If\, A \in \mathcal{A}, then\, A^c \coloneqq \Omega \backslash \mathrm{A} \in \mathcal{A}$
- $If\, A_1, A_2, \ldots \in \mathcal{A}, then \cup_{j=1}^{\infty} A_j \in \mathcal{A}$

The elements of $\mathcal{A}$ are called events!

Probability Theory

Probability Measure (formal definition): Let $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ be a sigma algebra. A map $\mathbb{P}: \mathcal{A} \to [0,1]$ is called a probability measure if:

- $\mathbb{P}(\Omega) = 1$, and $\mathbb{P}(\emptyset) = 0$.
- $\mathbb{P}\left(\cup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} \mathbb{P}(A_j)$ if sets A_i and A_j are pairwise disjoint.

Conditional Probability: For a given probability space $(\Omega, \mathcal{A}, \mathbb{P})$, given $B \in \mathcal{A}$ with $\mathbb{P}(B) \neq 0$, then, the conditional probability of A under B is

$$\mathbb{P}(A|B) \coloneqq \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

Durrett, Rick. Probability: theory and examples. Vol. 49. Cambridge university press, 2019.

Random Variable

Goal: To put all the relevant information of a random experiment into one object.

Random Variable: Let $(\Omega, \mathcal{A})$ and $(\tilde{\Omega}, \tilde{\mathcal{A}})$ be measurable spaces. A map $X: \Omega \to \tilde{\Omega}$ is called a random variable if $X^{-1}(\tilde{A}) \in \mathcal{A}$ for all $\tilde{A} \in \tilde{\mathcal{A}}$.

Some important notation: Let $(\Omega, \mathcal{A})$ and $(\tilde{\Omega}, \tilde{\mathcal{A}})$ be measurable spaces. Then,
$\mathbb{P}(X \in \tilde{A}) := \mathbb{P}\left(X^{-1}(\tilde{A})\right) = \mathbb{P}(\{\omega \in \Omega \mid X(\omega) \in \tilde{A}\})$.

Distribution: Given a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, and $X: \Omega \to \mathbb{R}$ be a random variable. Then, $\mathbb{P}_X: \mathcal{B}(\mathbb{R}) \to [0,1]$ defined by $\mathbb{P}_X(\tilde{A}) := \mathbb{P}\left(X^{-1}(\tilde{A})\right) = \mathbb{P}(X \in A)$ is called the probability distribution of X.

Some important notation: If $\widetilde{\mathbb{P}}$ is a probability measure and $\mathbb{P}_X = \widetilde{\mathbb{P}}$, then we say $X \sim \widetilde{\mathbb{P}}$.

Durrett, Rick. Probability: theory and examples. Vol. 49. Cambridge university press, 2019.

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- Probability Theory
- Risk Measures
- Coherent Risk Measures
- Distributionally Robust Risk Measures

What is risk?

Let us consider our investment as has a loss (or profit) distribution, which can be represented by a random variable, i.e., X.

- One way of looking at the risk is how badly the loss is going to be, e.g., the expected loss.
- Or, with some certain level of confidence ε, the loss is going to be less than some certain value with probability $1 - \varepsilon$

Let us also consider a platoon of two cars driving closely to each other with their inter-vehicle distance is a random variable d, and the unwanted event is the inter-vehicle collision.

- Then the risk is how closely both cars are going to experience the inter-vehicle collision.

Risk Measures: VaR

Value at Risk (VaR) : For a given random variable X which takes values in $\mathbb{R}$, the VaR at level $\varepsilon \in (0,1)$ is defined as:

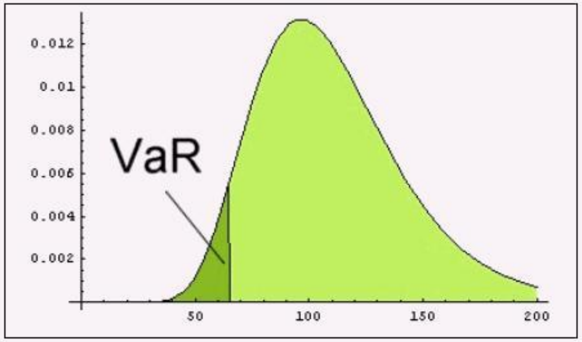
$$VaR_\varepsilon(X) = \inf\{x \in \mathbb{R} \mid \mathbb{P}(X > x) < \varepsilon\}$$

or

$$VaR_\varepsilon(X) = \inf\{x \in \mathbb{R} \mid \mathbb{P}(X < x) > 1 - \varepsilon\}$$

This risk measure is epically suitable for random variables that obtain continuous probability distributions, and it describes the expected loss given certain confidence level.

VaR does not control scenarios exceeding the VaR



https://analytica.com/risk-management-and-var-not-safe-for-everybody/

Risk Measures: VaR

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For normally distributed random variables, VaR is proportional to the standard deviation.

If $X \sim \mathcal{N}(\mu, \sigma^2)$ and $F_X(z)$ is the cumulative distribution function of X, then,

$$VaR_{1-\varepsilon}(X) = F_X^{-1}(1-\varepsilon) = \mu + k(1-\varepsilon)\,\sigma$$

where $k(1-\varepsilon) = \sqrt{2}\,\mathrm{erf}^{-1}(1-2\varepsilon)$ and $\mathrm{erf}(z) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^z e^{-t^2}\,dt$.

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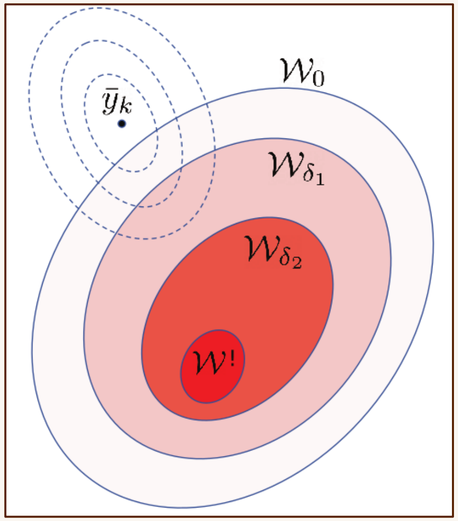
Uryasev, Stan, et al. "VaR vs CVaR in risk management and optimization." CARISMA conference. 2010.

Risk Measures: Systemic Sets

In the case that the undesired event is not in a continuous manner, e.g., inter-vehicle collision, the risk measures can still be established by defining a systemic set of the undesired event.

Let us assume the set of undesirable values of the system is given by U. Then, we can define a collection of systemic sets, U_δ, parametrized by $\delta \in [0, \infty]$. The systemic set is defined in the manner that it enjoys the following properties:

- $U_{\delta_1} \subset U_{\delta_2}$ when $\delta_1 > \delta_2$.
- $\lim_{n\to\infty} U_{\delta_n} = \bigcap_{n=1}^{\infty} U_{\delta_n} = U$ for any sequence $\{\delta_n\}_{n=1}^{\infty}$ with $\lim_{n\to\infty} \delta_n = \infty$.



Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

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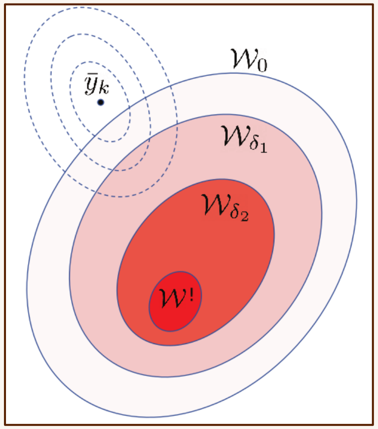
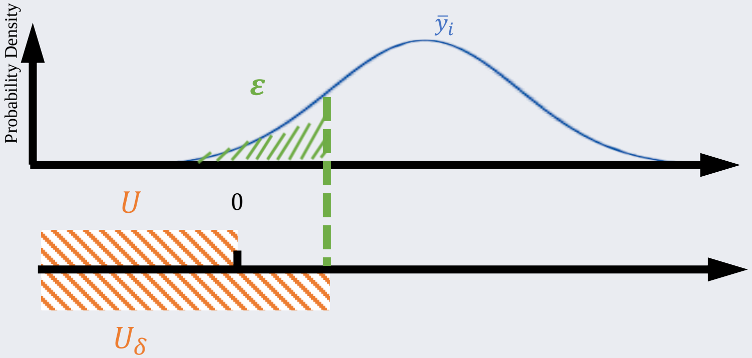


Design a collection of systemic sets for this problem that satisfies the above conditions

$$U = (\infty, 0)$$

$$U_\delta = ?$$



$$U_\delta = (\infty, \frac{c_1}{\delta + c_2})$$

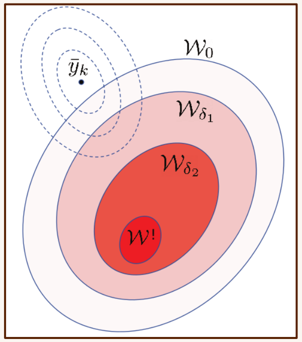
Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Risk Measures: VaR with systemic sets

Then, for a real-valued random variable y with probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we define the systemic event as $\{y \in U\}$, and the VaR is defined as follows.

Value at Risk: (New definition using systemic sets) For a given random variable y, the VaR at level $\varepsilon \in (0,1)$ is defined as:

$$VaR_\varepsilon(y) = \inf\{\, \delta > 0 \mid \mathbb{P}(y \in U_\delta) < \varepsilon \}.$$



The parameter $\varepsilon \in (0,1)$ denotes the level of confidence in the systemic events (e.g., inter-vehicle collision). The smaller this value, the higher the confidence of the random variable y stays away from the systemic set U.

The value-at-risk measure, VaR, represents the intuitive notion of "risk." The higher its value, the higher chance the system will be steered into the undesirable ranges of values.

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Risk Measures: AV@R

Conditional Value at Risk (CVaR) / Average Value at Risk (AV@R) /Expected Shortfall:

For a given random variable X, the AV@R at level $\varepsilon \in (0,1)$ is defined as:

$$AV@R_{\varepsilon} = \int_{-\infty}^{+\infty} z \, dF_X^{1-\varepsilon}$$

where

$$F_X^{1-\varepsilon}(z) = \begin{cases} 0 & when\ z < VaR_{1-\varepsilon}(X) \\ \dfrac{F_X(Z) - 1 + \varepsilon}{\varepsilon} & when\ z \geq VaR_{1-\varepsilon}(X) \end{cases}.$$

- AV@R is continuous with respect to α
- AV@R is convex in X

Uryasev, Stan, et al. "VaR vs CVaR in risk management and optimization." CARISMA conference. 2010.

Risk Measures: AV@R

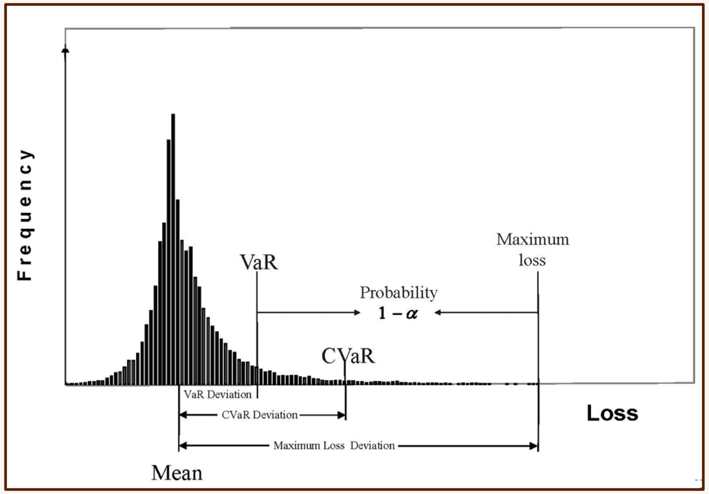
Some equivalent definition of AV@R for better understanding:

Optimization:

$$AV@R_\alpha(X) = \inf_c \{c + \frac{1}{1-\alpha}\mathbb{E}[X-c]^+\}$$

where

$$[X-c]^+ = \begin{cases} 0 & if\ X \le c \\ X - c & if\ X > c \end{cases}$$

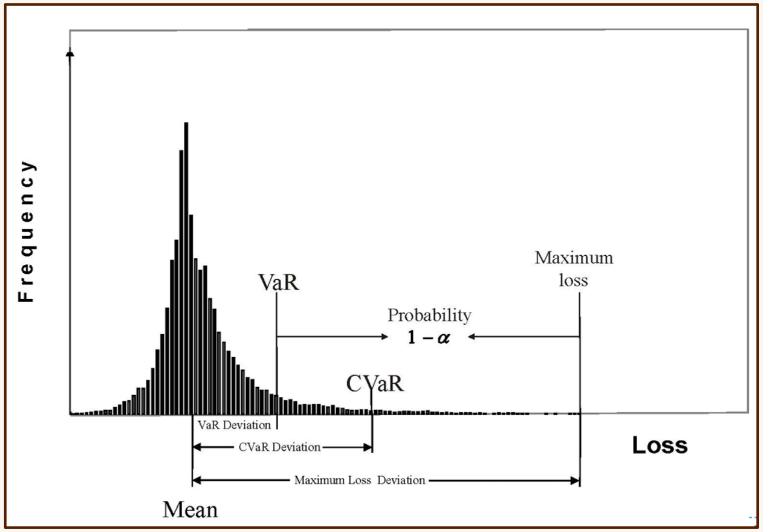


Expected Shortfall: $AV@R_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_\beta(X)\,d\beta.$

Expected shortfall is calculated by averaging all of the returns in the distribution that are worse than the VAR of the portfolio at a given level of confidence.

Pflug, Georg Ch. "Some remarks on the value-at-risk and the conditional value-at-risk." Probabilistic constrained optimization: Methodology and applications (2000): 272-281.
Acerbi, Carlo. "Spectral measures of risk: A coherent representation of subjective risk aversion." Journal of Banking & Finance 26.7 (2002): 1505-1518.

Risk Measures: AV@R



- AV@R has superior mathematical properties versus VaR
- AV@R accounts for losses exceeding VaR, i.e., it captures the severity of the failure
- AV@R deviation is a strong competitor to the Standard Deviation

Uryasev, Stan, et al. "VaR vs CVaR in risk management and optimization." CARISMA conference. 2010.

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Coherent Risk Measures

Some properties of risk measures:

Translation Invariance: For all X, and every constant $a \in \mathbb{R}$, the risk measure ρ satisfies

$$\rho(X + a) = \rho(X) + a.$$

Subadditivity: For all X_1 and X_2, the risk measure ρ satisfies

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2).$$

Positive Homogeneity: For all X, and every $\lambda > 0$ the risk measure ρ satisfies

$$\rho(\lambda X) \leq \lambda\, \rho(X).$$

Monotonicity: For $X_1 \leq X_2$ almost surely, the risk measure ρ satisfies

$$\rho(X_1) \leq \rho(X_2).$$

Coherent Risk Measures

Coherent Risk Measure: The risk measure ρ is called coherent if it satisfies the translation invariance, subadditivity, positive homogeneity, and monotonicity. Otherwise, it is incoherent.

Are those risk measures coherent?

- VaR: No. VaR is not sub-additive.
- AV@R: Yes.

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What's the next step?

In most real-world applications, the probability measure (density) of the uncertainty is unknown or inaccurate.

Ambiguity Set: We aim to focus on a certain set of probability measures that lies within certain distance to a target probability measure

$$\mathfrak{M} = \{\mathbb{Q} \mid d(\mathbb{P}, \mathbb{Q}) \leq r\}$$

Wasserstein Metric: For any $p \in [1, \infty)$, the type-p Wasserstein distance between two probability measures $\mathbb{Q}$ and $\mathbb{Q}'$ on $\mathbb{R}^m$ is defined as

$$W_p(\mathbb{Q}, \mathbb{Q}') = \left(\inf_{\pi \in \prod(\mathbb{Q}, \mathbb{Q}')} \int_{\mathbb{R}^m \times \mathbb{R}^m} \|\xi - \xi'\|^p \, \pi(d\xi, d\xi') \right)^{\frac{1}{p}}$$

where $\prod(\mathbb{Q}, \mathbb{Q}')$ denotes the set of all joint probability measures of ξ and ξ' with marginals $\mathbb{Q}$ and $\mathbb{Q}'$.

Wasserstein Metric

Wasserstein Metric: For any $p \in [1, \infty)$, the type-p Wasserstein distance between two probability measures $\mathbb{Q}$ and $\mathbb{Q}'$ on $\mathbb{R}^m$ is defined as

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where $\prod(\mathbb{Q}, \mathbb{Q}')$ denotes the set of all joint probability measures of ξ and ξ' with marginals $\mathbb{Q}$ and $\mathbb{Q}'$.

Example: For two normal distributions with equal means, the type-2 Wasserstein metric is given by

$$W(\Sigma_1, \Sigma_2) := \sqrt{\mathrm{Tr}[\Sigma_1] + \mathrm{Tr}[\Sigma_2] - 2\mathrm{Tr}\left[\sqrt{\sqrt{\Sigma_2}\Sigma_1\sqrt{\Sigma_2}}\right]}$$

Distributionally Robust Risk Measures

How should we construct a distributionally robust risk measure?

A. **Best-case Estimation** among all probability measures

B. **Worst-case Estimation** among all probability measures

C. **Average Estimation** among all probability measures

D. **Estimation of a randomly selected** probability measures

E. **Estimation of a User Specific** probability measure

F. I don’t know, let’s talk about it tomorrow

Distributionally Robust Risk Measures

Distributionally Robust Risk Measure: For a given random variable $X \in \mathbb{R}$ and the ambiguity set $\mathfrak{M}$, the distributionally robust risk measure is defined as

$$\rho(X) = \sup_{\mathbb{Q} \in \mathfrak{M}} \mathbb{E}^{\mathbb{Q}}[X]$$

Distributionally Robust Optimization: For a given random variable $X(\pi) \in \mathbb{R}$ and the ambiguity set $\mathfrak{M}$, the distributionally robust optimization problem is formulated as

$$J = \text{minimize}_{\pi \in \Pi} \sup_{Q \in \mathfrak{M}} \mathbb{E}^{\mathbb{Q}}[X(\pi)]$$

Some Useful References

- Durrett, Rick. Probability: theory and examples. Vol. 49. Cambridge university press, 2019.
- Uryasev, Stan, et al. "VaR vs CVaR in risk management and optimization." CARISMA conference. 2010.
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- Rahimian, Hamed, and Sanjay Mehrotra. "Distributionally robust optimization: A review." arXiv preprint arXiv:1908.05659 (2019).
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- Mohajerin Esfahani, Peyman, and Daniel Kuhn. "Data-driven distributionally robust optimization using the Wasserstein metric: Performance guarantees and tractable reformulations." Mathematical Programming 171.1-2 (2018): 115-166.
- Micheli, Francesco, Tyler Summers, and John Lygeros. "Data-driven distributionally robust MPC for systems with uncertain dynamics." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

ACC 2023 Workshop

Principles of Risk Quantification in Networked Control Systems

Risk Analysis in First-Order Consensus Network

Vivek Pandey

Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

9:15 am - 10:00 am

Section Overview

- Motivation
- Problem Statement
- Preliminary Result
- Risk of Large Fluctuation
- Risk of Cascading Large Fluctuation
- Fundamental Limits and Trade-offs
- Conclusions

Motivation: Why rendezvous?

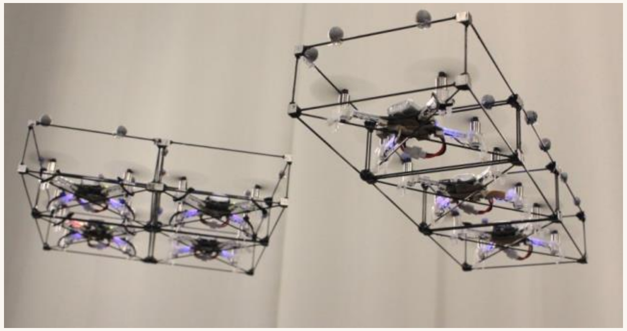
ren·dez·vous

Verb

meet at an agreed time and place.

"I **rendezvoused with** Bea as planned"

Rendezvous in time



Rendezvous in place

Saldana, David, et al. Modquad: The flying modular structure that self-assembles in midair. (ICRA 2018)

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Problem Statement: Rendezvous in Time

A team of n agents talk and decide when to meet. Their initial beliefs are given by $x_1(0), \dots, x_n(0)$ and they are updated as follows:

$$\mathrm{d}x_i(t) = u_i(t)\,\mathrm{d}t + b\,\mathrm{d}w_i(t),$$

$$u_i(t) = \sum_{j=1}^{n} k_{ij}\left(x_j(t-\tau) - x_i(t-\tau)\right)$$

Gaussian Noise

Time Delays

Communication Graph Structure

The input weight k_{ij} denotes how much each agent will trust the beliefs from the other agent. By collecting all the input weights, the closed loop dynamic can be converted into a compact form using the graph Laplacian matrix.

Problem Statement: Rendezvous in Time

Let's put it in a compact form, with L the graph Laplacian and $B = b\, I_n$,

$$\mathrm{d}\boldsymbol{x}_t = -L\, \boldsymbol{x}_{t-\tau}\, \mathrm{d}t + B\, \mathrm{d}\boldsymbol{w}_t.$$

The graph Laplacian matrix a defined element-wise as

$$(L)_{i,j} = \begin{cases} -k_{ij} & if\ i \neq j \\ \sum k_{ij} & if\ i = j \\ 0 & otherwise \end{cases}.$$

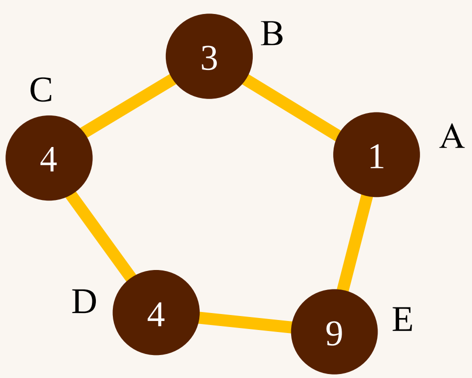
When the graph is connected, the eigenvalues of the Laplacian matrix L enjoys the following property:

- The smallest eigenvalue is zero with algebraic multiplicity one.
- The spectrum of L can be ordered as $0 = \lambda_1 \leq \cdots \leq \lambda_n$.
- The eigenvector corresponding to λ_k is q_k with $q_1 = \frac{1}{\sqrt{n}}$.
- $L = \mathrm{Q}\Lambda Q^T$, where $\mathrm{Q} = [q_1 | \cdots | q_n]$ is an orthogonal matrix and $\Lambda = \mathrm{diag}[0 | \lambda_2 | \cdots | \lambda_n]$

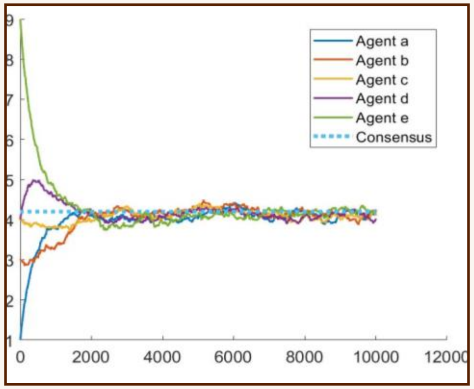
Problem Statement: Rendezvous in Time

In a big picture:

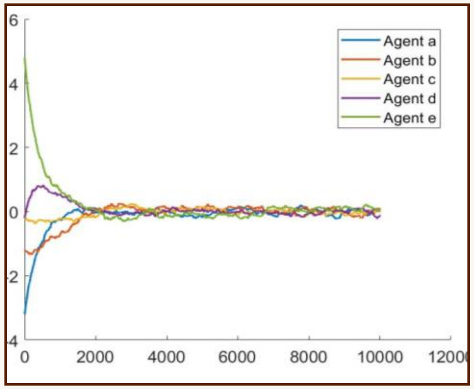
- A team of agents aim to meet at the same time.
- Each agent has its initial opinion/belief.
- They exchange and update their opinions via a communication network.
- There exists uncertainty and time-delay for the communication.



Agents and their initial beliefs



State vs time



Deviation vs time

Somarakis, C., Ghaedsharaf, Y. and Motee, N., 2019. Time-delay origins of fundamental tradeoffs between risk of large fluctuations and network connectivity. IEEE Transactions on automatic control, 64(9), pp.3571-3586.

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Preliminary Results: Conditions for Consensus

How do we know agents will reach the consensus? There are two assumptions.

Assumption 1: The communication graph is undirected and connected.

Assumption 2: The closed loop system is stable if and only if the time-delay satisfies $\tau < \frac{\pi}{2\lambda_n}$.

In absence of exogenous noise, the system reaches the consensus of $\frac{1}{n}\sum_{i=1}^{n} x_i(0)$ as $t \to \infty$.

Consequently, the exogenous noise excites the observable modes of the network, and the state fluctuates around the consensus.

Somarakis, C., Ghaedsharaf, Y. and Motee, N., 2019. Time-delay origins of fundamental tradeoffs between risk of large fluctuations and network connectivity. IEEE Transactions on automatic control, 64(9), pp.3571-3586.

Preliminary Results: Observables and their statistics

Observables: Deviation between agent's state and the current average

$$\boldsymbol{y}_t = M_n\, \boldsymbol{x}_t,$$

in which $M_n = I_n - \frac{1}{n} 1_n 1_n^T$ is the centering matrix, and $\boldsymbol{y}_t$ will oscillate around $\mathbf{0}$ in the steady-state.

Steady-state Statistics: When the network has reached the consensus, the steady-state statistics of $\bar{\boldsymbol{y}} = \boldsymbol{y}_\infty$ is shown by

$$\bar{\boldsymbol{y}} \sim \mathcal{N}(0, \Sigma),$$

And the elements of $\Sigma = [\sigma_{ij}]$ are shown by

$$\sigma_{ij} = \frac{1}{2} b^2 \sum_{k=2}^{n} \frac{\cos(\lambda_k \tau)}{\lambda_k (1 - \sin(\lambda_k \tau))} (\boldsymbol{m}_i^T \boldsymbol{q}_k)(\boldsymbol{m}_j^T \boldsymbol{q}_k),$$

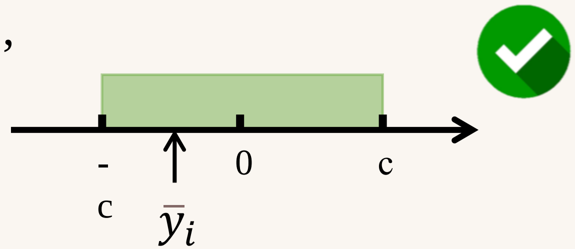
where m_i denotes the i-th column of M_n, and λ_k is the k-th eigenvalue of L.

Preliminary Results: What is the FAILURE?

C-consensus event: Since the observable $\bar{\boldsymbol{y}}$ fluctuates around 0, we allow some tolerance of the disagreement such that

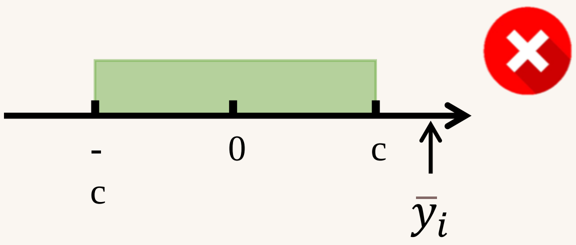
$$|\bar{\boldsymbol{y}}|_\infty \leq c,$$

which is also named as c-consensus event.



Large Fluctuation: The failure is considered as the i-th agent fails to reach the c-consensus such that

$$|\bar{y}_i| > c.$$



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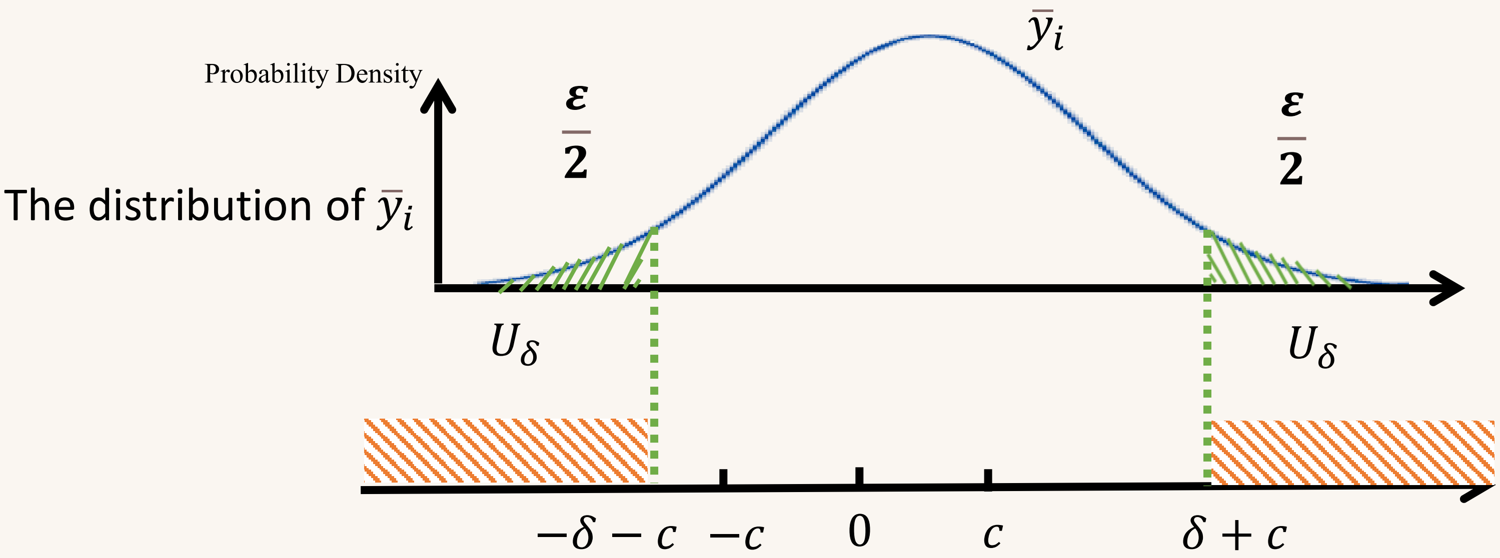
Somarakis, C., Ghaedsharaf, Y. and Motee, N., 2019. Time-delay origins of fundamental tradeoffs between risk of large fluctuations and network connectivity. IEEE Transactions on automatic control, 64(9), pp.3571-3586.

Preliminary Results: What is the RISK?

The value at risk measure is defined as

$$\mathcal{R}_\varepsilon = \inf \{ \delta > 0 | \mathbb{P}\{\bar{y}_i \in U_\delta\} < \varepsilon\}.$$



The confidence level $\varepsilon \in (0,1)$ and use it to find the systemic set $U_\delta = (-\infty, -\delta - c) \cup (\delta + c, \infty)$ with $U_\infty = U$.

The undesired set of values $U = \{-\infty\} \cup \{\infty\}$.

Rockafellar, R.T. and Uryasev, S., 2000. Optimization of conditional value-at-risk. Journal of risk, 2, pp.21-42.

Section Overview

- Motivation
- Problem Statement
- Preliminary Result
- Risk of Large Fluctuation
- Risk of Cascading Large Fluctuation
- Fundamental Limits and Trade-offs
- Conclusions

Risk of Large Fluctuation

Lemma 1: The conditional distribution of $\bar{y}_j$ follows a normal distribution $\mathcal{N}(\tilde{\mu}_j, \tilde{\sigma}_j^2)$

Theorem 1: The risk of large fluctuation of a single agent j is given by

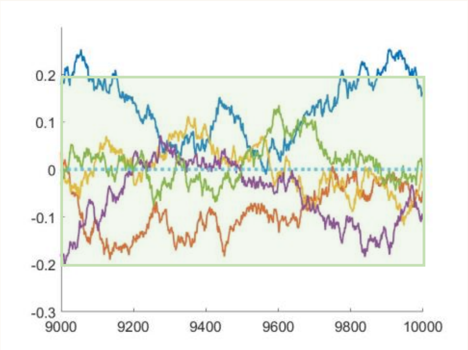
$$\mathcal{R}_\varepsilon^j = \sqrt{2}\sigma_j \iota_\varepsilon - c, \qquad if \quad \sigma_j > \frac{c}{\sqrt{2}\iota_\varepsilon},$$

where $\iota_\varepsilon = \operatorname{erf}^{-1}(1 - \varepsilon)$

Y. L. Tong. The multivariate normal distribution. Springer Science & Business Media, 2012.

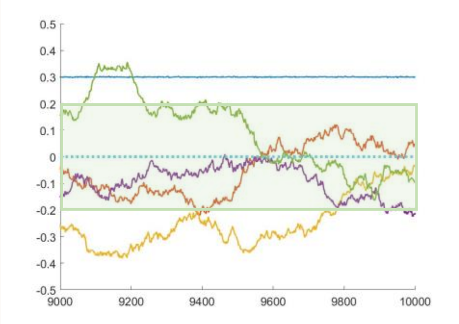
Risk of Cascading Failures: Why cascading failures?

In realistic systems the large fluctuation is inevitable even if we design control laws against them. And if the failure happens, designing for the “what now" is a good idea (e.g. cascading risk)



y_t vs time

We want our network to be able to isolate the existing failure and prevent the future failures.



y_t vs time

G. Liu, C. Somarakis, and N. Motee. “Risk of Cascading Failures in Time-Delayed Vehicle Platooning”. IEEE CDC (2021).

M. Rahnamay-Naeini and M. M. Hayat. “Cascading Failures in Interdependent Infrastructures: An Interdependent Markov-Chain Approach”. In: IEEE Transactions on Smart Grid 7.4 (2016)

Y. Zhang and O. Ya˘gan. “Robustness of interdependent cyber-physical systems against cascading failures”. In: IEEE Transactions on Automatic Control 65.2 (2019)

Risk of Cascading Failures: Conditional Distribution

We construct the cascading failure by considering the conditional distribution of j-th agent when some agent has failed to reach the c-consensus, e.g., $\bar{y}_j \mid |\bar{y}_i| > c$.

Lemma 2: The conditional distribution of $\bar{y}_j \mid |\bar{y}_i| = y_f > c$ follows a normal distribution $\mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2)$ such that

$$\tilde{\mu} = \rho_{ij} \frac{\sigma_j}{\sigma_i} y_f, \qquad \tilde{\sigma}^2 = \sigma_j^2 (1 - \rho_{ij}^2) ,$$

where $\rho_{ij} = \sigma_{ij} / \sigma_i \, \sigma_j$, and $|\rho_{ij}| < 1$.

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Y. L. Tong. The multivariate normal distribution. Springer Science & Business Media, 2012.

Risk of Cascading Failures: Conditional Distribution

We construct the cascading failure to rendezvous by considering the conditional distribution of the j −th agent when some agents with ordered indices $\mathcal{I}_m = \{i_1, \dots, i_m\}$ with $j \notin \mathcal{I}_m$ for some $m < n - 1$ have failed to rendezvous, i.e., $\bar{\boldsymbol{y}}_{\mathcal{I}_m} = \boldsymbol{y}_f$.

Let us form a 2×2 block matrix in $\mathbb{R}^{(m+1)\times(m+1)}$

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} \\ \tilde{\Sigma}_{21} & \tilde{\Sigma}_{22} \end{bmatrix},$$

where $\tilde{\Sigma}_{11} = \sigma_j^2$, $\tilde{\Sigma}_{12} = \tilde{\Sigma}_{21}^T = [\sigma_{j,i_1}, \dots, \sigma_{j,i_m}]$, and $\tilde{\Sigma}_{22} = \left[\sigma_{k_1,k_2}\right]_{k_1,k_2 \in \mathcal{I}_m} \in \mathbb{R}^{m\times m}$.

Lemma 3: The conditional distribution of $\bar{y}_j | \bar{\boldsymbol{y}}_{\mathcal{I}_m}$ follows a multivariate normal distribution $\mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2)$ such that

$$\tilde{\mu} = \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}(\boldsymbol{y}_f), \qquad \tilde{\sigma}^2 = \tilde{\Sigma}_{11} - \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}.$$

Y. L. Tong. The multivariate normal distribution. Springer Science & Business Media, 2012.

Risk of Cascading Failures

In the view of failure to reach consensus, we define the event of under the risk of failure for $\bar{y}_j$ as

$$U_\delta = (-\infty, -\delta - c) \cup (\delta + c, \infty) \text{ with } U_\infty = U.$$

for $\delta \in [0, \infty]$ and $c \geq 1$. The risk of cascading failure is measured by assuming the i'th (or $\mathcal{I}_m = \{i_1, \ldots, i_m\}$) agents have failed to reach consensus, i.e.,

$$\mathcal{R}_\varepsilon^{i,j} = \inf \{ \delta > 0 \mid \mathbb{P}\{\bar{y}_j \in U_\delta \mid |\bar{y}_i| = y_f\} < \varepsilon\}$$

or

$$\mathcal{R}_\varepsilon^{\mathcal{I}_m,j} = \inf \{ \delta > 0 \mid \mathbb{P}\{\bar{y}_j \in U_\delta \left| \bar{\boldsymbol{y}}_{\mathcal{I}_m} \right| = \boldsymbol{y}_f\} < \varepsilon\}$$

with the confidence level $\varepsilon \in (0,1)$.

G. Liu, C. Somarakis, and N. Motee. “Risk of Cascading Failures in Time-Delayed Vehicle Platooning”. IEEE CDC (2021).

Risk of Cascading Failures: Single Existing Failure

Theorem 2: Suppose the network reaches the steady-state and the i-th agent has failed to reach the consensus with the observable $|\bar{y}_i| = y_f$. The risk of cascading large fluctuation at the j-th agent is

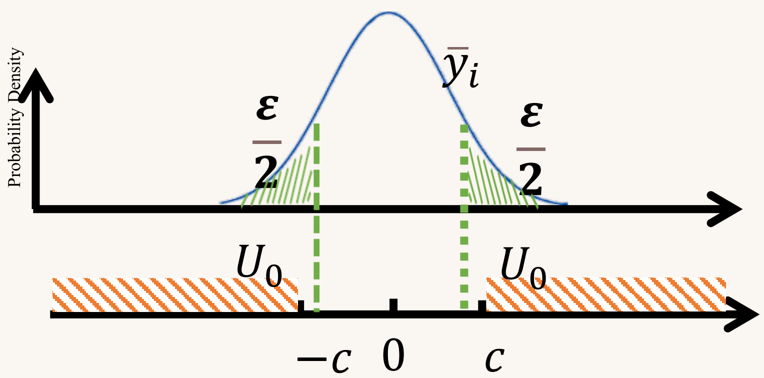
$$\mathcal{R}_{\varepsilon}^{i,j} := \begin{cases} 0, & \text{if} \quad 1 - \frac{1}{2}\left(\operatorname{erf}\left(\kappa_{0,+}^{i,j}\right) + \operatorname{erf}\left(\kappa_{0,-}^{i,j}\right)\right) \leq \varepsilon \\ S(\delta), & \text{otherwise} \end{cases}$$

$$\kappa_{\delta,\pm}^{i,j} = \frac{(\delta + c)\sigma_i^2 \pm \sigma_{ij} y_f}{\sigma_i \sqrt{2(\sigma_i^2 \sigma_j^2 - \sigma_{ij}^2)}}$$

$$S(\delta) = \inf\left\{\delta > 0 \,|\, \operatorname{erf}\left(\kappa_{\delta,+}^{i,j}\right) + \operatorname{erf}\left(\kappa_{\delta,-}^{i,j}\right) > 2(1 - \varepsilon)\right\}$$

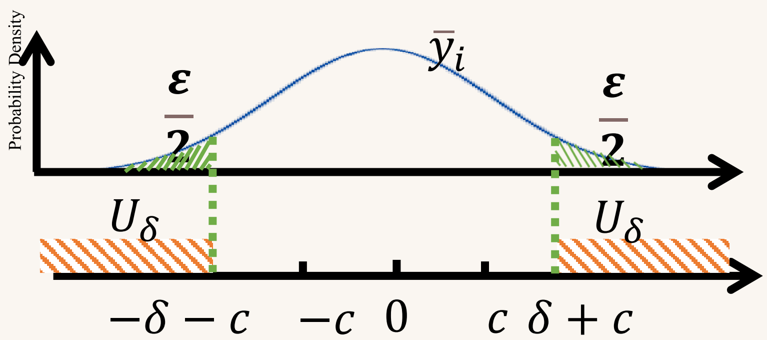
Risk of Cascading Failures: Single Existing Failure

$$0, \quad \text{if} \quad 1 - \frac{1}{2}\left(\text{erf}\left(\kappa_{0,+}^{i,j}\right) + \text{erf}\left(\kappa_{0,-}^{i,j}\right)\right) \leq \varepsilon$$



A narrow distribution or a low confidence level

$$S(\delta), \quad \text{otherwise}$$



A wide distribution or a high confidence level

Risk of Cascading Failures: Multiple Existing Failure

Theorem 3: Suppose the network reaches the steady-state and the agents with indices $\mathcal{I}_m = \{i_1, \dots, i_m\}$ have failed to reach the consensus with the observable $|\bar{\boldsymbol{y}}_{\mathcal{I}_m}| = \boldsymbol{y}_f$. The risk of cascading large fluctuation at the j-th agent is

$$\mathcal{R}_{\varepsilon}^{\mathcal{I}_m,j} := \begin{cases} 0, & \text{if} \quad 1 - \frac{1}{2}\left(\text{erf}\left(\kappa_{0,+}^{\mathcal{I}_m,j}\right) + \text{erf}\left(\kappa_{0,-}^{\mathcal{I}_m,j}\right)\right) \leq \varepsilon \\ \mathrm{S}(\delta), & \text{otherwise} \end{cases}$$

$$\kappa_{\delta,\pm}^{i,\mathcal{I}_m} = \frac{(\delta + c) \pm \tilde{\mu}}{\sqrt{2}\tilde{\sigma}}$$

$$\mathrm{S}(\delta) = \inf\left\{\delta > 0 \mid \text{erf}\left(\kappa_{\delta,+}^{\mathcal{I}_m,j}\right) + \text{erf}\left(\kappa_{\delta,-}^{\mathcal{I}_m,j}\right) > 2(1-\varepsilon)\right\}$$

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Update Law for Computation of Cascading Risk

We consider the scenatio where agents with labels $\mathcal{I}_m$ are found in failure states and we aim to update the statistics of the agent of interest, i.e., $\bar{y}_j | \bar{\boldsymbol{y}}_{\mathcal{I}_m} = \boldsymbol{y}_f$, when a new failure at agent $k \notin \mathcal{I}_m$ is discovered.

Let us consider the following notations

$$\tilde{\mu}_j = \tilde{\Sigma}_{12}(j)\tilde{\Sigma}_{22}^{-1}(\boldsymbol{y}_f), \qquad \tilde{\sigma}_j^2 = \sigma_j^2 - \tilde{\Sigma}_{12}(j)\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}(j),$$

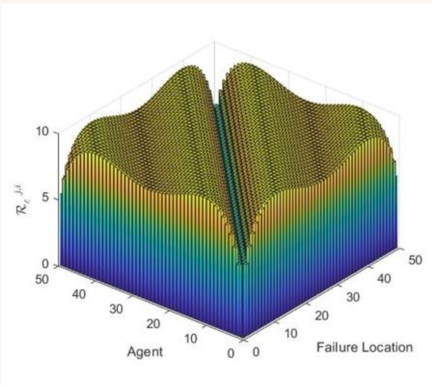
$$\tilde{\mu}_k = \tilde{\Sigma}_{12}(k)\tilde{\Sigma}_{22}^{-1}(\boldsymbol{y}_f), \qquad \tilde{\sigma}_k^2 = \sigma_k^2 - \tilde{\Sigma}_{12}(k)\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}(k),$$

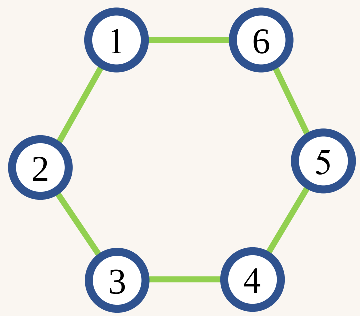
Theorem 4: Suppose that $\bar{y}_j$ follows $\mathcal{N}(\tilde{\mu}_j, \tilde{\sigma}_j^2)$ when m agents have already failed with label with label $\mathcal{I}_m$. The updated conditional distribution $\bar{y}_j$ when a new agent fails, i.e., agent $k \notin \mathcal{I}_m$ with observable $|y_{f_k}| > c$, $\mathcal{N}(\tilde{\mu}', \tilde{\sigma}'^2)$ such that

$$\tilde{\mu}' = \tilde{\mu}_j - \frac{\tilde{\sigma}_{jk}}{\tilde{\sigma}_k^2}(\tilde{\mu}_k - y_{f_k}), \qquad \tilde{\sigma}'^2 = \tilde{\sigma}_j^2 - \frac{\tilde{\sigma}_{jk}}{\tilde{\sigma}_k^2},$$

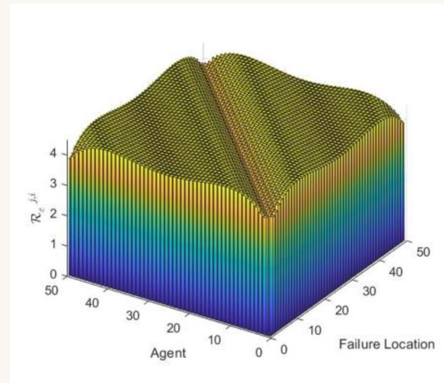
where $\tilde{\sigma}_{jk} = \sigma_{jk} - \tilde{\Sigma}_{12}(k)\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}(j).$

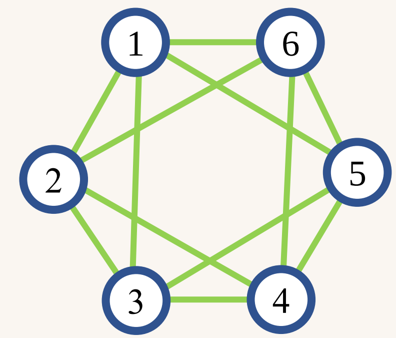
Risk of Cascading Failures: Single Existing Failure



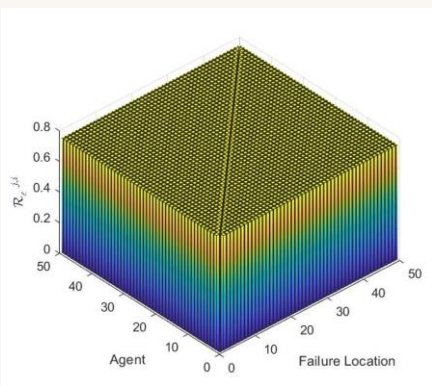


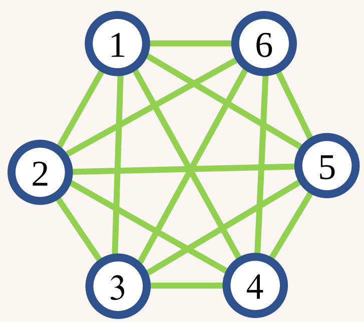
1-Cycle Graph



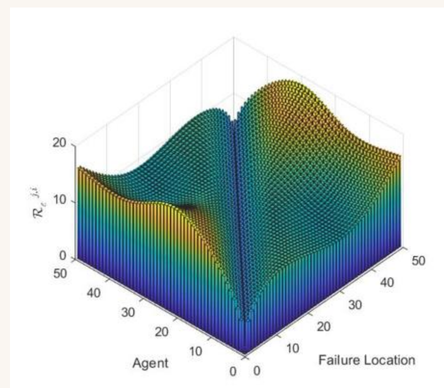


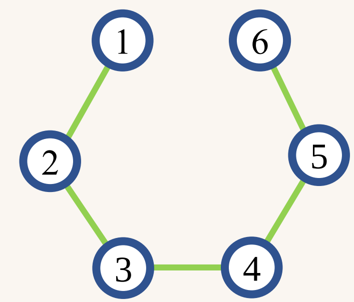
2-Cycle Graph





Complete Graph





Path Graph

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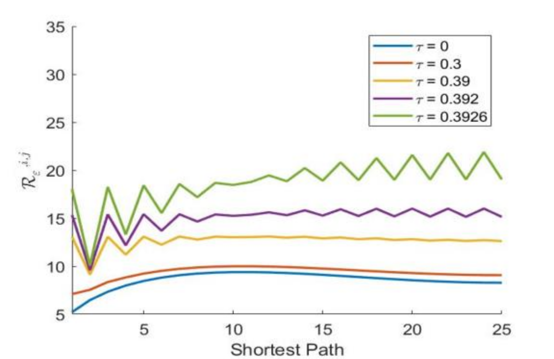
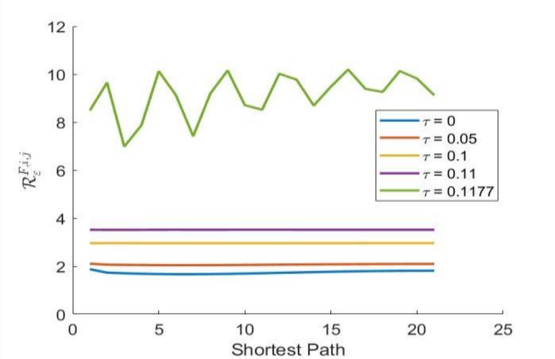
Risk of Cascading Failures: Shortest Path: Single Existing Failure



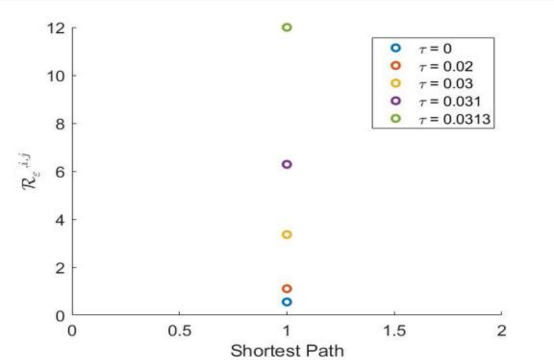
1-cycle graph

Increasing trend



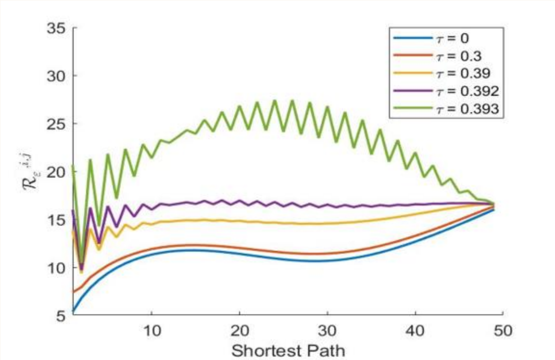
5-cycle graph

Less increasing trend



Complete graph

No trend
Same distance



Path graph

Trend depends on the time-delay τ

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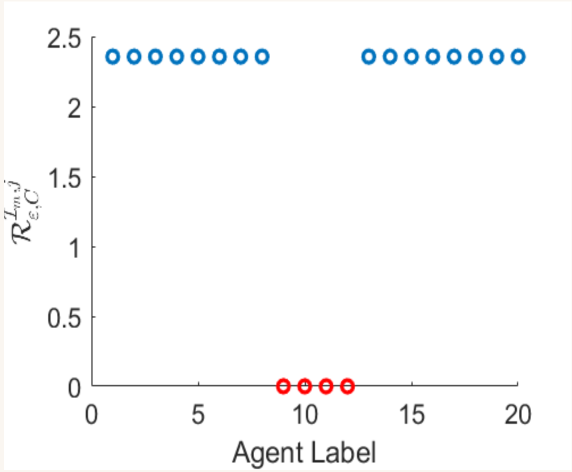
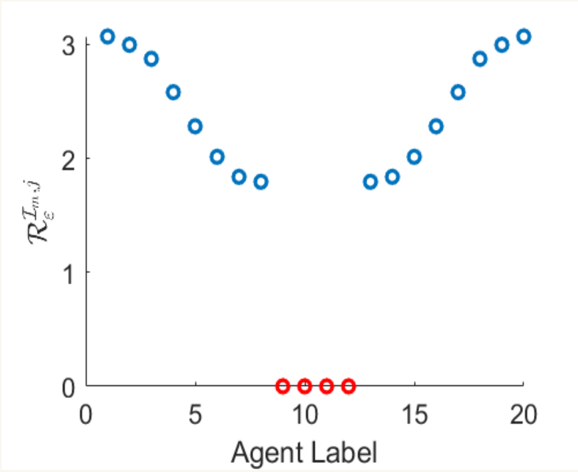
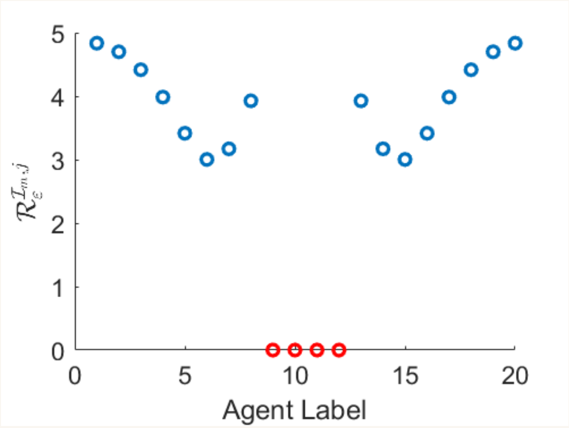
Risk of Cascading Failures: Multiple Existing Failures



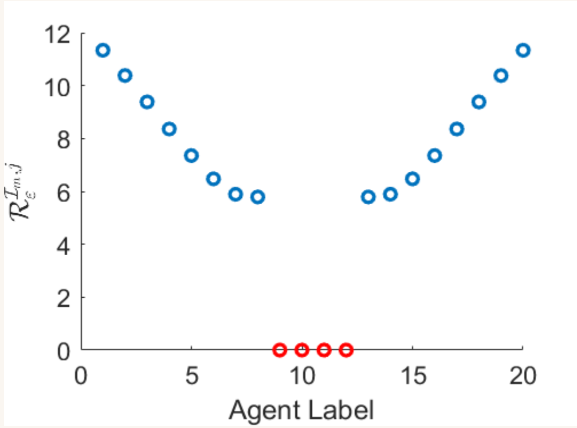
Complete Graph



5–cycle graph



2-cycle graph



Path Graph

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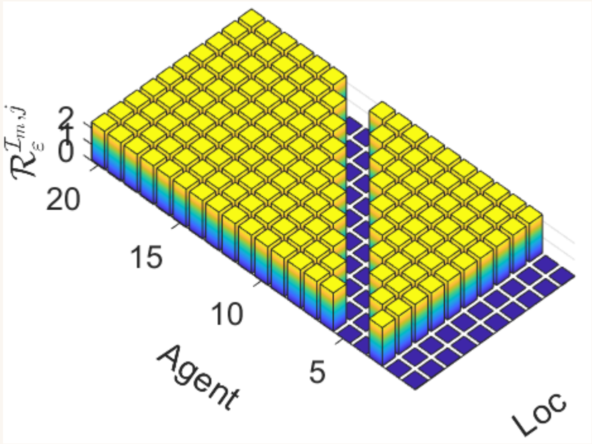
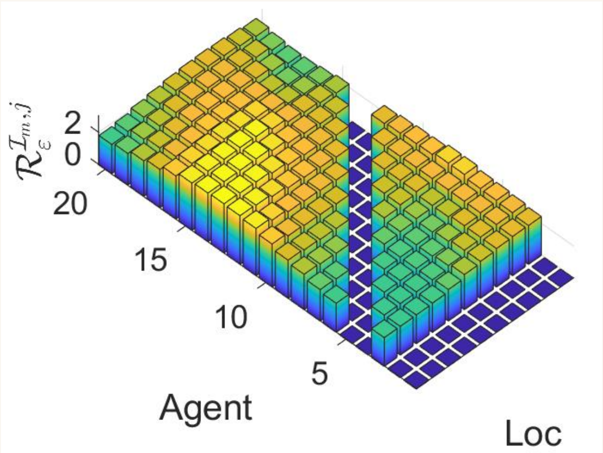
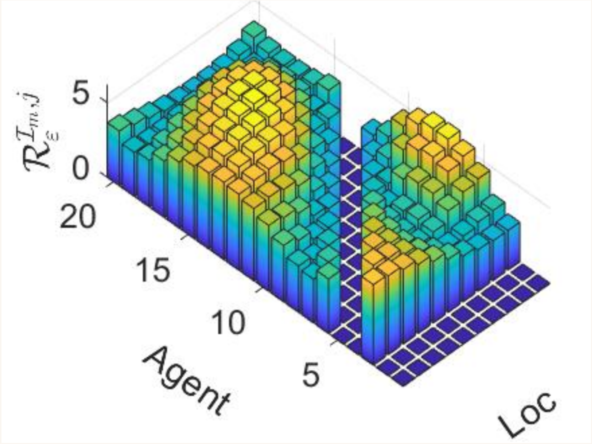
Risk of Cascading Failures: Multiple Existing Failures



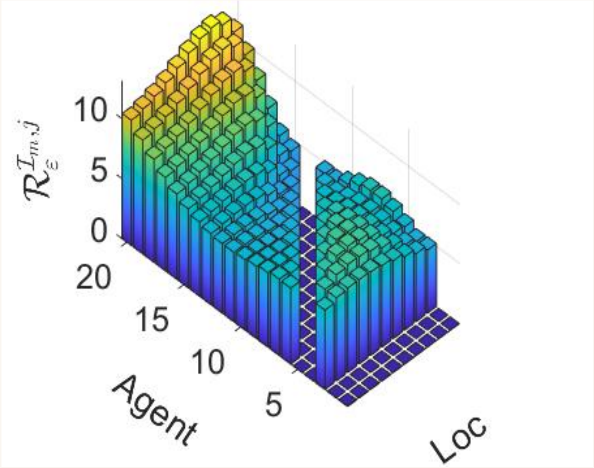
Complete Graph



5–cycle graph



2-cycle graph



Path Graph

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Fundamental Limits and Trade-offs

Lemma 4: For a team of agents adopting the complete graph with their steady-states observables $\bar{\boldsymbol{y}} \sim \mathcal{N}(0, \Sigma)$, the elements of its covariance matrix Σ is shown by

$$\sigma_{ij} := \begin{cases} \frac{n-1}{2n^2} \frac{\cos(n\tau) b^2}{1-\sin(n\tau)}, & \text{if } i = j \\ -\frac{1}{2n^2} \frac{\cos(n\tau) b^2}{1-\sin(n\tau)}, & \text{if } i \neq j \end{cases}$$

Lemma 5: For the steady-state statistics of the observables $\bar{\boldsymbol{y}}$, the diagonal elements of its covariance matrix Σ satisfies the lower bound

$$\sigma_i \geq \sqrt{\frac{n-1}{n} b^2 \tau \underline{f}} = \sigma^*,$$

with $\underline{f} = 1.52$, the lower bound of f.

Fundamental Limits and Trade-offs

Theorem 5: In a complete communication graph, there exists a fundamental limit on the cascading large fluctuation.

$$\mathcal{R}_{\varepsilon}^{i,j} \geq \begin{cases} 0, & \text{if} \quad 1 - \frac{1}{2}\left(\text{erf}\left(\zeta_{0,+}^{*}\right) + \text{erf}\left(\zeta_{0,-}^{*}\right)\right) \leq \varepsilon \\ \text{S}^{*}(\delta), & \text{otherwise} \end{cases}$$

$$\zeta_{\delta,\pm}^{*} = \frac{(n-1)(\delta + c) \pm y_f}{\sigma^{*}\sqrt{2n(n-2)}}$$

$$\text{S}^{*}(\delta) = \inf\left\{ \delta > 0 \,|\, \text{erf}\left(\zeta_{\delta,+}^{*}\right) + \text{erf}\left(\zeta_{\delta,-}^{*}\right) > 2(1-\varepsilon)\right\}$$

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Conclusions

- Value-at-risk framework of cascading systemic failures.
- Risk profile of cascading failures is quantified using the steady-state statistics obtained from the system observables.
- The cascading risk quantifies the impact from the existing failures on the consensus network.
- Time-delayed fundamental limit on special graph structures.

ACC 2023 Workshop

Principles of Risk Quantification in Networked Control Systems

Risk Analysis in Second-Order Consensus Network: Autonomous Vehicle Platooning

Guangyi Liu

Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

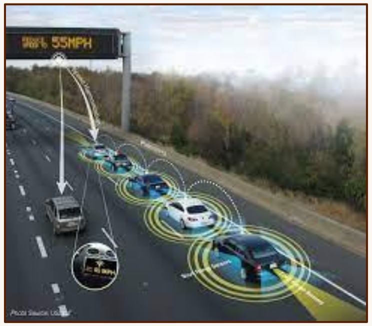
10:30 am -11:25 am

Section Overview

- Motivation
- Problem Statement
- Preliminary Result
- Risk of Inter-vehicle Collision
- Fundamental limitations and trade-offs
- Risk of Cascading Inter-vehicle Collision
- Conclusions

platooning

In transportation, platooning or flocking is a method for driving a group of vehicles together. It is meant to increase the capacity of roads via an automated highway system.



https://www.wikiwand.com/en/Platoon_%28automobile%29

https://pnorental.com/truck-platooning-the-future-of-road-transport/

https://www.c4isrnet.com/2022/08/10/us-army-lethality-task-force-looks-to-ai-to-decrease-casualties/

Problem Formulation: Vehicle Platooning

What conditions need to be satisfied to form a platoon?

A. Time-invariant inter-vehicle distance

B. Same velocity

C. Converge to the steady-state

D. ☺

E. The velocity needs to be positive

F. All the inter-vehicle distances must be the same

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Problem Formulation: Vehicle Platooning

A team of n self-driving vehicles communicates to others and aim to form a platoon with a constant velocity and inter-vehicle distance. For the i'th vehicle, its position and velocity is shown by $x_t^{(i)}$ and $v_t^{(i)}$. And the vehicle-wise dynamics is governed by

$$\mathrm{d}x_t^{(i)} = v_t^{(i)}\mathrm{d}t,$$
$$\mathrm{d}v_t^{(i)} = u_t^{(i)}\mathrm{d}t + g\,\mathrm{d}\xi_t^i,$$

Brownian motions

where the control input $u_t^{(i)}$ is given by

$$u_t^{(i)} = \sum_{j=1}^{n} k_{i,j}\left(v_{t-\tau}^{(j)} - v_{t-\tau}^{(i)}\right) + \beta \sum_{j=1}^{n} k_{i,j}\left(x_{t-\tau}^{(j)} - x_{t-\tau}^{(i)} - (j-i)r\right)$$

Time Delays

Time Delays

Communication Graph Structure

Problem Formulation: Vehicle Platooning

Let's put it in a compact form, with L denotes the Laplacian matrix of the communication graph

$$\mathrm{d}\boldsymbol{x}_t = \boldsymbol{v}_t \, \mathrm{d}t,$$

$$\mathrm{d}\boldsymbol{v}_t = -L \, \boldsymbol{v}_{t-\tau} \, \mathrm{d}t - \beta L(\boldsymbol{x}_{t-\tau} - \boldsymbol{r})\mathrm{d}t + g\mathrm{d}\, \boldsymbol{\xi}_{\boldsymbol{t}},$$

where $\boldsymbol{x}_t = \left[x_t^{(1)}, x_t^{(2)}, \dots, x_t^{(n)}\right]^T$ and $\boldsymbol{v}_t = \left[v_t^{(1)}, v_t^{(2)}, \dots, v_t^{(n)}\right]^T$ are collections of positions and velocities of vehicles, $\boldsymbol{r} = [r, 2r, \dots, nr]^T$ is the vector of target inter-vehicle distances.

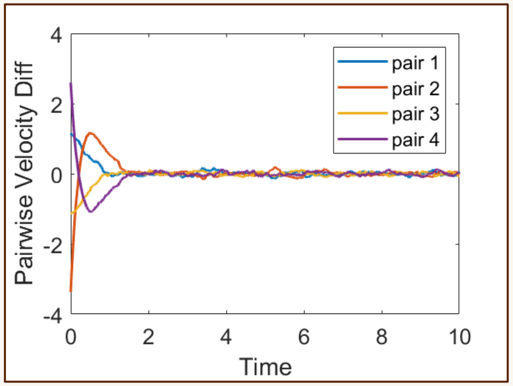
Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Problem Formulation: Vehicle Platooning

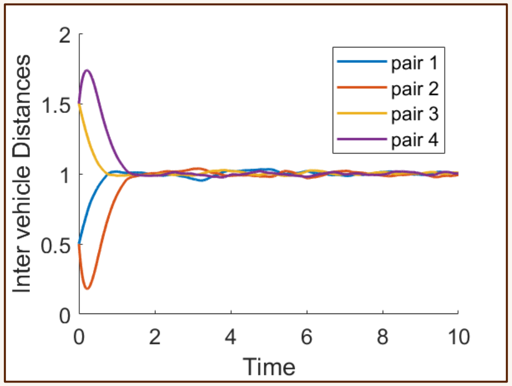
In a big picture:

- A team of self-driving vehicles aim to form a platoon.
- The platoon has constant inter-vehicle distance and velocity.
- They exchange and update their states via a communication network.
- There exists uncertainty and time-delay for the communication and the control input.

Car platoon with the complete communication graph



Pairwise velocity diff vs time



Inter-vehicle distance vs time

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

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Preliminary Results: Steady-State and Stability Conditions

The steady-state of the platoon with $g = 0$ as when

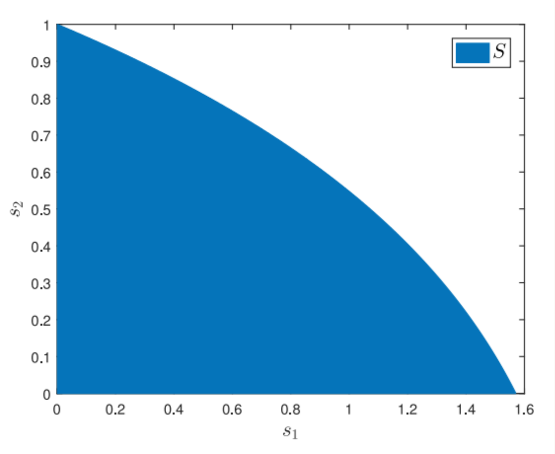
$$\lim_{t\to\infty} \left|v_t^{(j)} - v_t^{(i)}\right| = 0 \text{ and } \lim_{t\to\infty} \left|x_t^{(j)} - x_t^{(i)} - (i-j)\, r\right| = 0,$$

for all i, j and initial conditions.

The afore mentioned noise-free consensus network will converge and form the platoon if and only if $(\lambda_i \tau, \beta\tau) \in S$ for all $i = 2, \dots, n$, where

$$S = \left\{(s_1, s_2) \in \mathbb{R}^2 \middle| s_1 \in \left(0, \frac{\pi}{2}\right), s_2 \in \left(0, \frac{a}{\tan(a)}\right)\right\},$$

with $a \in \left(0, \frac{\pi}{2}\right)$ the solution of $a\sin(a) = s_1$, and λ_i is the i'th eigenvalue of the graph Laplacian L in the non-decreasing order.



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Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Preliminary Results: Steady-state Inter-Vehicle Distance

Consequently, the exogenous noise excites the steady-state observable modes of the network, and the state fluctuates around the consensus.

Observables: In order to ensure the safety of the platoon, let us consider the observable as the (steady-state) inter-vehicle distances, such that

$$\bar{d}_i := \lim_{t\to\infty} (x_t^{(i+1)} - x_t^{(i)})$$

whenever it exists. The collection of the inter vehicle distances is shown by $\bar{\boldsymbol{d}} = [\bar{d}_i, \dots, \bar{d}_i]^T \in \mathbb{R}^{n-1}$.

Steady-state Statistics: Once the network has reached the consensus, the steady-state inter-vehicle distance $\bar{\boldsymbol{d}}$ is proven to be a random vector in $\mathbb{R}^{n-1}$ and it follows a multi-variate normal distribution, such that

$$\bar{\boldsymbol{d}} \sim \mathcal{N}(r\mathbf{1}_{n-1}, \Sigma)$$

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Preliminary Results: Steady-state Inter-Vehicle Distance

Steady-state Statistics: The steady-state inter-vehicle distance vector $\bar{\boldsymbol{d}} \sim \mathcal{N}(r\mathbf{1}_{n-1}, \Sigma)$ has a mean of the target platoon distance r and its covariance matrix $\Sigma = [\sigma_{i,j}]$ is shown element-wise by

$$\sigma_{i,j} = g^2 \frac{\tau^3}{2\pi} \sum_{k=2}^{n} \left(\tilde{\boldsymbol{e}}_i^T \boldsymbol{q}_k\right)\left(\tilde{\boldsymbol{e}}_j^T \boldsymbol{q}_k\right) f(\lambda_k \tau, \beta\tau),$$

for all $i, j = 1, \dots, n-1$ and

$$f(s_1, s_2) = \int_{\mathbb{R}} \frac{d\, r}{(s_1 s_2 - r^2 \cos(r))^2 + r^2 (s_1 - r \sin(r))^2}.$$

In the expression above, λ_k denotes the k'th eigenvector of L, $\boldsymbol{q}_k$ denotes its corresponding normalized eigenvector, and $\tilde{\boldsymbol{e}}_i$ is given by $\boldsymbol{e}_{i+1} - \boldsymbol{e}_i$.

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Section Overview

- Motivation
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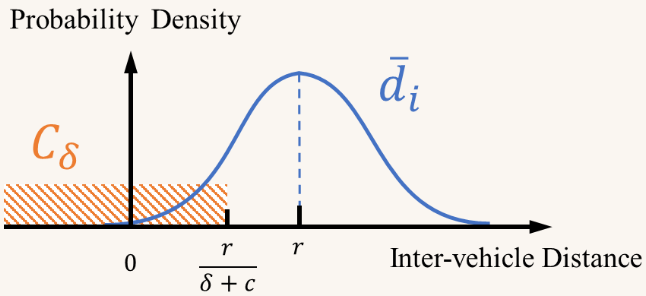
- Conclusions

FAILURE? RISK?

Inter-vehicle Collision: In this work, we consider the event of failure as the inter-vehicle collision, which is given by

$$\{\bar{d}_i \in (-\infty, 0)\}.$$

Level sets and Value-at-Risk Measure: A family of level sets $C_\delta = (-\infty, \frac{r}{\delta+c})$ helps to construct an alarm zone that describes how vehicles are dangerously close to the collision. The Value-at-Risk measure is an effective tool to quantify the chance of failure by evaluating



$$\mathcal{R}_\varepsilon \coloneqq \inf\{\delta \geq 0 \mid \mathbb{P}\{\bar{d}_i \in C_\delta\} < \varepsilon\}.$$

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

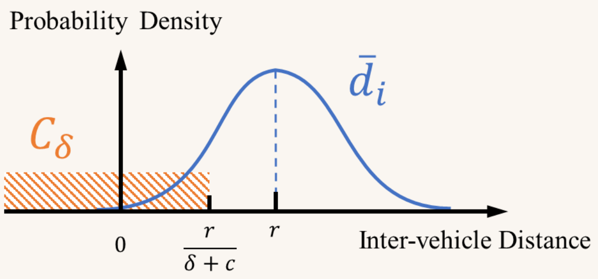
Risk of Inter-vehicle Collision

Suppose that the network of vehicles form a platoon in the steady-state. For every $i = 1, \dots, n-1$, the risk of inter-vehicle collision is

$$\mathcal{R}_\varepsilon^i = \begin{cases} 0 & \text{if } \sigma_i \le \dfrac{r}{\kappa_\varepsilon\sqrt{2}}\dfrac{c-1}{c} \text{ or } \varepsilon \ge \dfrac{1}{2} \\ \dfrac{r}{r-\kappa_\varepsilon\sigma_i\sqrt{2}} - c & \text{if } \dfrac{r}{\kappa_\varepsilon\sqrt{2}}\dfrac{c-1}{c} < \sigma_i < \dfrac{r}{\kappa_\varepsilon\sqrt{2}} \\ \infty & \text{if } \sigma_i \ge \dfrac{r}{\kappa_\varepsilon\sqrt{2}} \end{cases}$$

where $\kappa_\varepsilon := \text{erf}^{-1}(1-2\varepsilon) > 0$.

- For a large enough r , the inter-vehicle collision is unlikely to occur.
- When σ_i exceeds the ε dependent cutoff, the risk is ∞ since the collision can not be avoided with probability higher than $1-\varepsilon$.



Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

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Time-delay induced fundamental limits

Facts: An engineer has almost no control over the communication time-delay and exogenous disturbances.

Question: Can I still design an optimal communication topologies to minimize the risk in the networked control system?

Short answer: Somehow yes. There exists some communication graph topologies that can reduce the risk, but there also exists a fundamental limit on the best achievable risk.

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Time-delay induced fundamental limits

The marginal standard deviation σ_i satisfy the lower bound

$$\sigma_i = \sqrt{\sigma_{i,i}} \geq \sigma^* := \sqrt{\pi \underline{f}}\, |g|\, \tau^{\frac{3}{2}}$$

for all $i = 1, \ldots, n-1$, where $\underline{f} := \inf_{(s_1, s_2) \in S} f(s_1, s_2) \approx 25.4603.$

Key steps:

- f is nonnegative over S
- f obtains a minimum ($\underline{f}$) inside S
- $\sum_{k=2}^{n} (\tilde{\boldsymbol{e}}_i^T \boldsymbol{q}_k)(\tilde{\boldsymbol{e}}_i^T \boldsymbol{q}_k) = \|\tilde{\boldsymbol{e}}_i^T\|^2 = 2$

$$\sigma_{i,j} = g^2 \frac{\tau^3}{2\pi} \sum_{k=2}^{n} (\tilde{\boldsymbol{e}}_i^T \boldsymbol{q}_k)(\tilde{\boldsymbol{e}}_j^T \boldsymbol{q}_k) f(\lambda_k \tau, \beta \tau),$$

Independent to communication graph topologies!!!

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Time-delay induced fundamental limits

Then, the previous result can be immediately applied to the risk of inter-vehicle collisions.

There is an inherent fundamental limit on the best achievable values of risk of inter-vehicle collision in the platoon that is given by

$$\mathcal{R}^i_\varepsilon \geq \begin{cases} 0 & \text{if } \sigma^* \leq \frac{r}{\kappa_\varepsilon\sqrt{2}} \frac{c-1}{c} \text{ or } \varepsilon \geq \frac{1}{2} \\ \frac{r}{r - 4.02\kappa_\varepsilon |g| \tau^{3/2}} - c & \text{if } \frac{r}{\kappa_\varepsilon\sqrt{2}} \frac{c-1}{c} < \sigma^* < \frac{r}{\kappa_\varepsilon\sqrt{2}} \\ \infty & \text{if } \sigma^* \geq \frac{r}{\kappa_\varepsilon\sqrt{2}} \end{cases}$$

For any feasible τ and g, the optimal communication topology is a complete graph with link weights $k_{i,j} = \frac{s_1}{n\tau}$ for all $i, j = 1, \ldots, n$.

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Trade-offs

In order to reduce the risk of inter-vehicle collision to the extreme, how should we alter the communication graph connectivity?

A. Increase the connectivity as much as possible
B. Decrease the connectivity as much as possible
C. Increase the connectivity, but only to some extent
D. Decrease the connectivity, but only to some extent
E. I don't know, maybe ask chatgpt

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

Trade-offs

In addition to the fundamental limits of the risk, there also exists a counter-intuitive trade-off between the risk of collision and the network connectivity.

Effective Resistance: For a given communication graph, the effective resistance is defined as

$$\Xi_{\mathcal{G}} = n \sum_{i=2}^{n} 1/\lambda_i$$

The smaller the value of $\Xi_{\mathcal{G}}$, the stronger the connectivity of $\mathcal{G}$.

The best achievable level of risk of inter-vehicle collision and the communication connectivity emerges as follows

$$\mathcal{R}_{\varepsilon}^{i} \cdot \sqrt{\Xi_{\mathcal{G}}} > \sqrt{n\tau \underline{E} \left(\frac{2(n-1)}{\pi} + \sum_{m=1}^{\infty} \alpha_m \right)}$$

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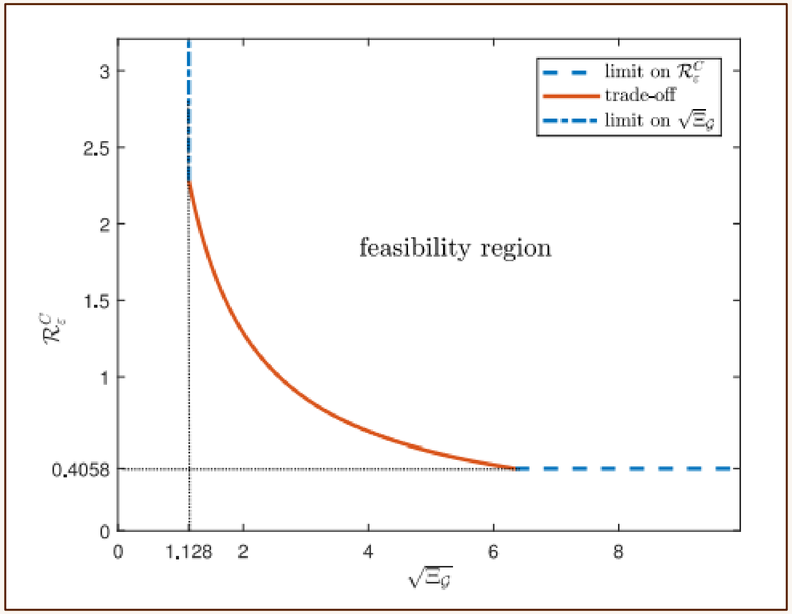
Trade-offs



For a nontrivial range of network parameters, the only way to maintain a safer (low-risk) network is trough weakening the communication connectivity, e.g., by decreasing the feedback gain between vehicles or sparsifying the communication graph.

Strengthening the connectivity of the network also increases the risk of inter-vehicle collision between vehicles.

Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

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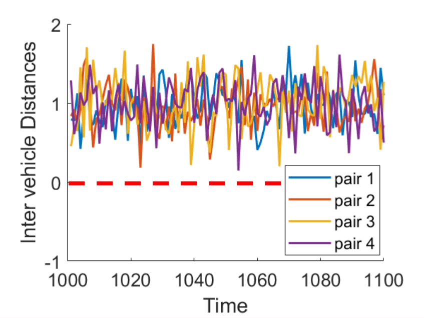
Risk of Cascading Failures: Why cascading failures?

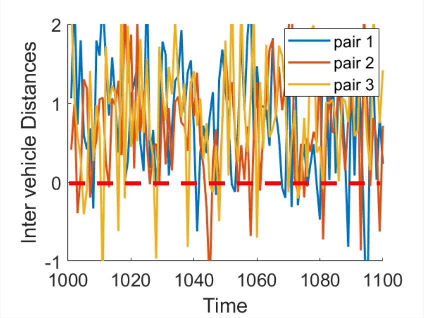
In real world platoons, the inter-vehicle collision is inevitable even if we design control laws against it. When the collision occurs, instead of asking "what if", we should design for the goal of "even if".

https://tenor.com/search/domino-gifs

We want our network to be able to isolate the existing failure and prevent the future failures.



Distances when no collision



Distances when pair 4 has collided

As collisions may cascade, there may exist more than one failures, and one needs to quantify the likelihood such cascade is going to occur.

G. Liu, C. Somarakis, and N. Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning". IEEE CDC (2021).

Risk of Cascading Failures: Conditional Distribution

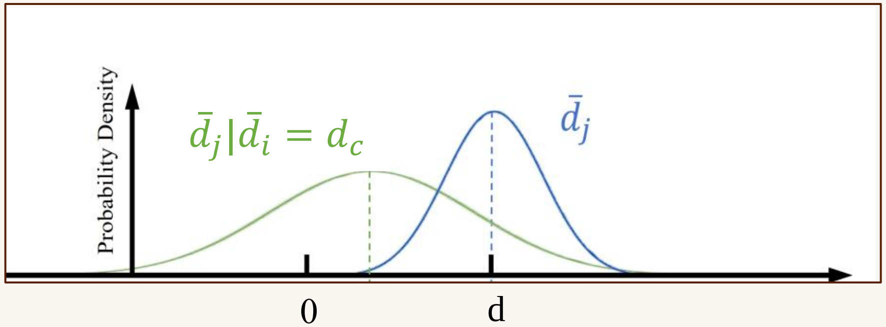
In order to evaluate the impact from one system failure to the other inter-vehicle distance, we investigate how it will change the distribution.

Given one pair of vehicle $\bar{d}_i$ has encountered the systemic failure with inter-vehicle distance of d_c, the conditional distribution of $\bar{d}_j|\bar{d}_i = d_c$ is given by $\mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2)$ with

$$\tilde{\mu} = r + \rho_{ij}\frac{\sigma_j}{\sigma_i}(d_c - r) \quad \text{and} \quad \tilde{\sigma}^2 = \sigma_j^2(1 - \rho_{ij}^2)$$

where $\rho_{ij} = \sigma_{i,j}/\sigma_i\sigma_j$ and $|\rho_{i,j}| < 1$.

The situation of inter-vehicle collision can be interpreted as $d_c = 0$.



Y. L. Tong. The multivariate normal distribution. Springer Science & Business Media, 2012.

Risk of Cascading Failures: Conditional Distribution

In the case of multiple existing collisions, we measure the risk of cascading collisions by considering the conditional distribution of the j'th pair when some pairs of vehicles with ordered indices $\mathcal{I}_m = \{i_1, \ldots, i_m\}$ with $j \notin \mathcal{I}_m$ for some $m < n-1$ have collided, i.e., $\bar{d}_{i_m} = 0$.

Let us form a 2×2 block matrix in $\mathbb{R}^{(m+1)\times(m+1)}$

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} \\ \tilde{\Sigma}_{21} & \tilde{\Sigma}_{22} \end{bmatrix},$$

where $\tilde{\Sigma}_{11} = \sigma_j^2$, $\tilde{\Sigma}_{12} = \tilde{\Sigma}_{21}^T = [\sigma_{j,i_1}, \ldots, \sigma_{j,i_m}]$, and $\tilde{\Sigma}_{22} = \left[\sigma_{k_1,k_2}\right]_{k_1,k_2 \in \mathcal{I}_m} \in \mathbb{R}^{m\times m}$.

The conditional distribution of $\bar{d}_j | \bar{\boldsymbol{d}}_{\mathcal{I}_m} = \mathbf{0}$ follows a multivariate normal distribution $\mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2)$ such that

$$\tilde{\mu} = r + \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}(-r\,\mathbf{1}_m), \qquad \tilde{\sigma}^2 = \tilde{\Sigma}_{11} - \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}.$$

G. Liu, C. Somarakis, and N. Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning". IEEE CDC (2021).

Risk of Cascading Collision

In the view of inter-vehicle collisions, we define the event of under the risk of collision for $\bar{d}_j$ as

$$\{\bar{d}_i \in C_\delta\} \text{ with } C_\delta = \left(-\infty, \frac{d}{\delta + c}\right)$$

for $\delta \in [0, \infty]$ and $c \geq 1$. The risk of cascading collision is measured by assuming the i'th pair (or $\mathcal{I}_m = \{i_1, \ldots, i_m\}$) of vehicles has collided, i.e.,

$$\mathcal{R}_\varepsilon^{i,j} = \inf \left\{\delta > 0 \mid \mathbb{P}\{\bar{d}_j \in C_\delta | \bar{d}_i = 0\} < \varepsilon\right\}$$

or

$$\mathcal{R}_\varepsilon^{\mathcal{I}_m,j} = \inf \left\{\delta > 0 \mid \mathbb{P}\{\bar{d}_j \in C_\delta | \bar{\mathbf{d}}_{\mathcal{I}_m} = 0\} < \varepsilon\right\}$$

with the confidence level $\varepsilon \in (0,1)$.

G. Liu, C. Somarakis, and N. Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning". IEEE CDC (2021).

Risk of Cascading Collision: Single existing collision

Suppose that the conditions of stability hold, and the i'th pair has collided. The risk of cascading inter-vehicle collision at the j'th pair is

$$\mathcal{R}_{\varepsilon}^{i,j} = \begin{cases} 0 & \text{if } \kappa_0^{(i,j)} \leq \iota_\varepsilon \\ \dfrac{r\sigma_i}{\gamma(i,j,\varepsilon)} - c & \text{if } \iota_\varepsilon \in \left(\kappa_\infty^{(i,j)}, \kappa_0^{(i,j)}\right) \\ \infty & \text{if } \kappa_\infty^{(i,j)} \geq \iota_\varepsilon \end{cases}$$

where $\iota_\varepsilon = \mathrm{erf}^{-1}(2\varepsilon - 1)$,

$$\kappa_\delta^{(i,j)} := \frac{r}{\sqrt{2(1-\rho_{ij}^2)}\sigma_j}\left(\frac{1}{\delta + c} + \rho_{ij}\frac{\sigma_j}{\sigma_i} - 1\right) \quad \text{and} \quad \gamma(i,j,\varepsilon) = \iota_\varepsilon \sigma_i \sigma_j \sqrt{2(1-\rho_{ij}^2)} + r\sigma_i - r\rho_{ij}\sigma_j$$

G. Liu, C. Somarakis, and N. Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning". IEEE CDC (2021).

Risk of Cascading Collision: Single existing collision

When two pairs of vehicles are not correlated, i.e., ρ_{ij}, the risk of cascading collision can be reduced into the risk of single collision:

$$\begin{aligned}\gamma(i, j, \varepsilon) &= \iota_\varepsilon \sigma_i \sigma_j \sqrt{2(1-\rho_{ij}^2)} + r\sigma_i - r\rho_{ij}\sigma_j \\ &= \iota_\varepsilon \sigma_i \sigma_j \sqrt{2} + r\sigma_i\end{aligned}$$

then

$$\mathcal{R}_\varepsilon^{i,j} = \frac{r}{\gamma(i, j, \varepsilon)} - c = \frac{r}{\iota_\varepsilon \sigma_j \sqrt{2} + r} - c$$

G. Liu, C. Somarakis, and N. Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning". IEEE CDC (2021).

Risk of Cascading Collision: Multiple existing collisions

Suppose the platoon reaches the steady-state and vehicle pairs with label $\mathcal{J}_m$ have collided such that $\bar{\boldsymbol{d}}_{\mathcal{J}_m} = \mathbf{0}$. The risk of cascading collision at the j-th pair is

$$\mathcal{R}_{\varepsilon}^{\mathcal{J}_m, j} := \begin{cases} 0, & \text{if } \frac{r - c\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}\,c} \leq \iota_{\varepsilon} \\ \frac{r}{\sqrt{2}\,\iota_{\varepsilon}\tilde{\sigma} + \tilde{\mu}} - c, & \text{if } \iota_{\varepsilon} \in \left(\frac{-\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}}, \frac{r - c\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}\,c}\right) \\ \infty, & \text{if } \frac{-\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}} \geq \iota_{\varepsilon} \end{cases}$$

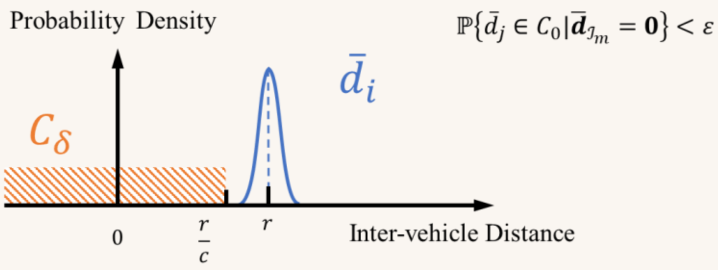
where $\iota_{\varepsilon} = \mathrm{erf}^{-1}(2\,\varepsilon - 1)$, and

$$\tilde{\mu} = r + \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}(-r\,\mathbf{1}_m), \qquad \tilde{\sigma}^2 = \tilde{\Sigma}_{11} - \tilde{\Sigma}_{12}\tilde{\Sigma}_{22}^{-1}\tilde{\Sigma}_{21}.$$

Risk of Cascading Collision

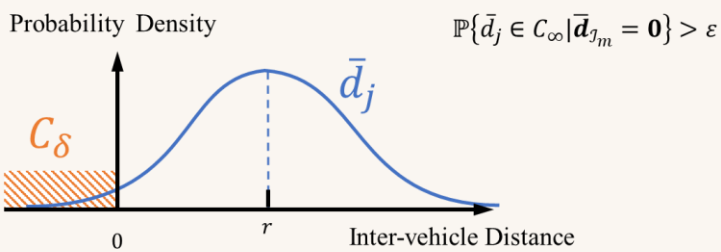
A narrow distribution or a low confidence level

$$0, \quad \text{if} \quad \frac{r - c\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}\,c} \leq \iota_\varepsilon$$

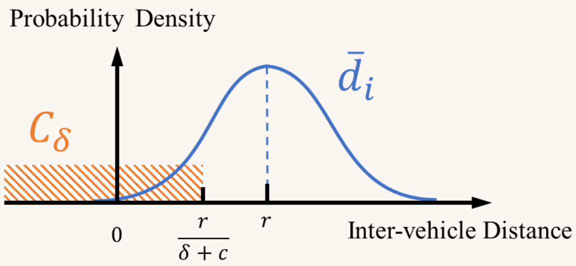


A wide distribution or a high confidence level

$$\infty, \quad \text{if} \quad \frac{-\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}} \geq \iota_\varepsilon$$



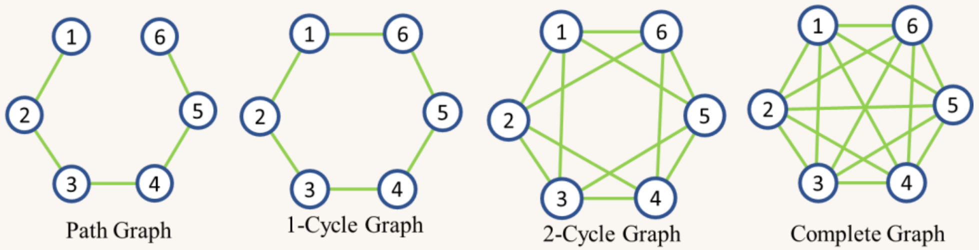
$$\frac{r}{\sqrt{2}\,\iota_\varepsilon\tilde{\sigma} + \tilde{\mu}} - c, \ \text{if} \quad \iota_\varepsilon \in \left(\frac{-\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}}, \frac{r - c\,\tilde{\mu}}{\sqrt{2}\,\tilde{\sigma}\,c}\right)$$



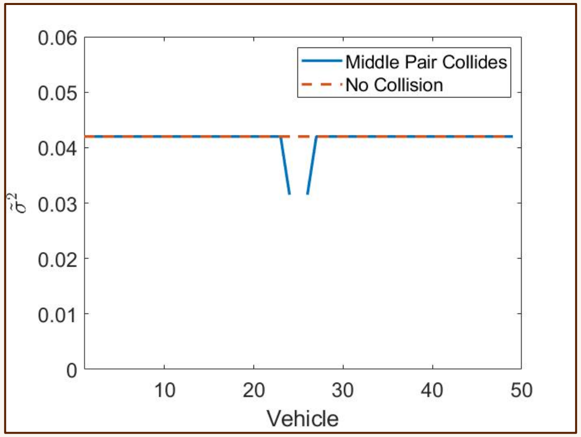
Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Risk of Cascading Collisions: Case Study

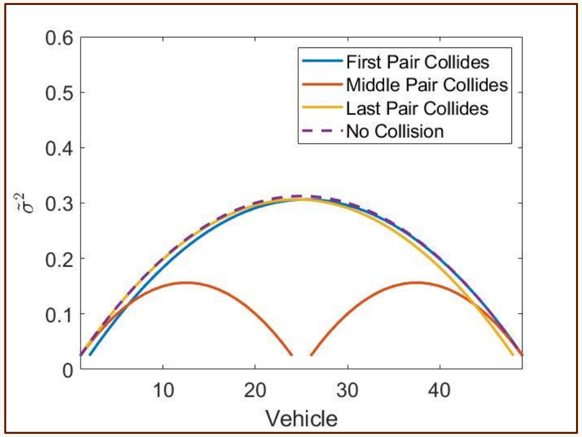
$n = 50$ vehicles aims to form a platoon with various communication graphs



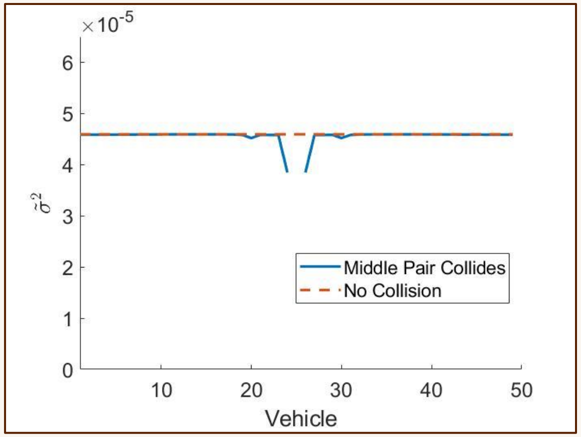
Change of the variance of the inter-vehicle distance



Complete Graph



Path Graph



5-cycle Graph

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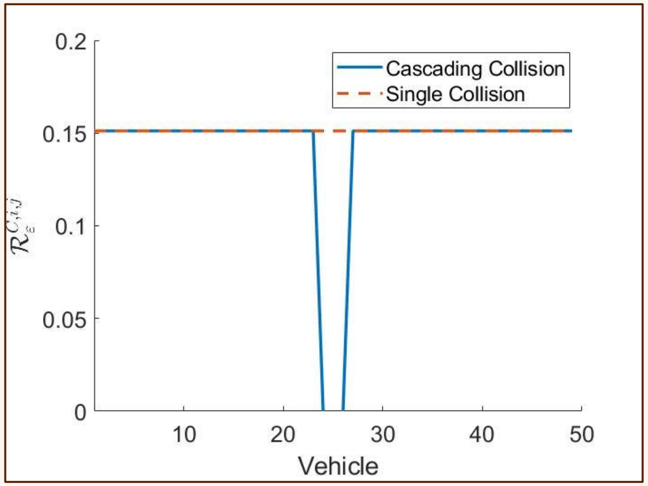
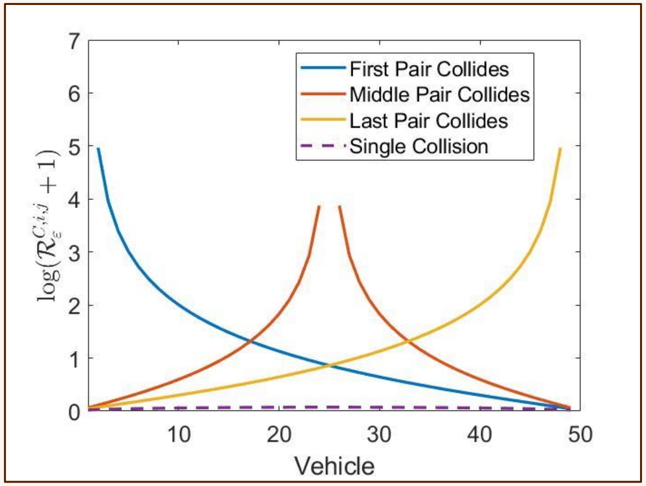
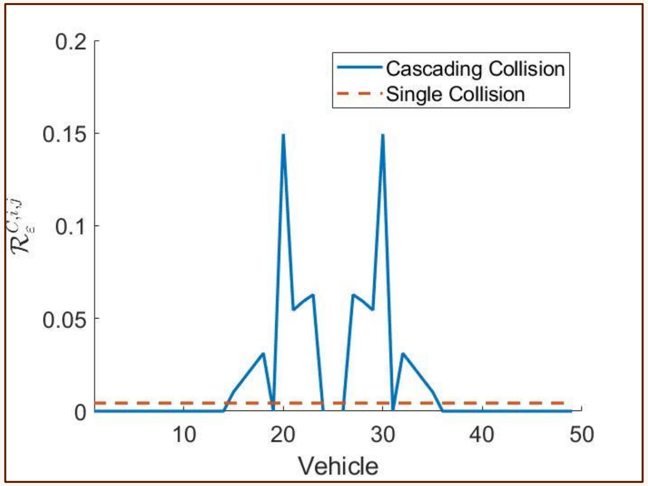
Risk of Cascading Collisions: Case Study



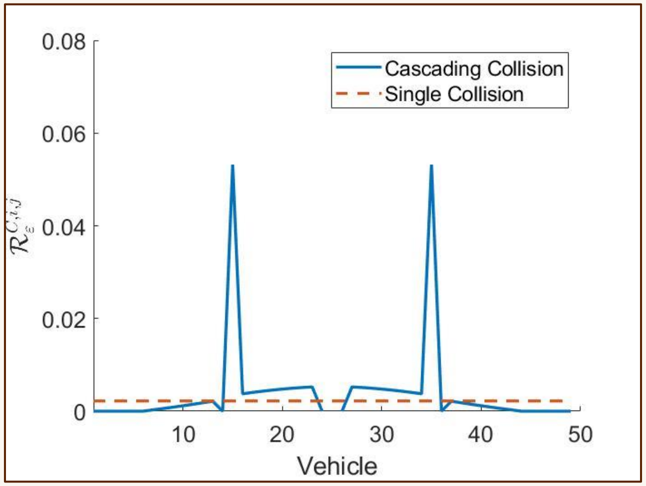
Complete Graph



Path Graph



5-cycle Graph



10-cycle Graph

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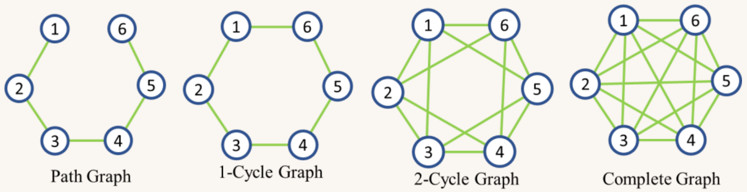
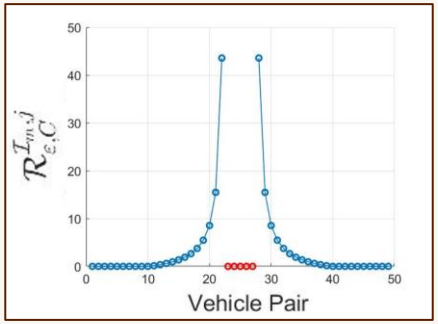
Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

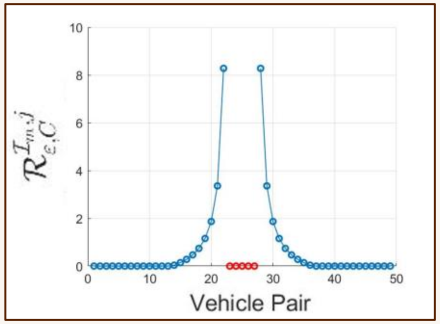
Risk of Cascading Collisions: Case Study

Risk profiles of a platoon with $n = 50$ vehicles, assuming pairs $\mathcal{I}_m = \{23,24,25,26,27\}$ have collided.

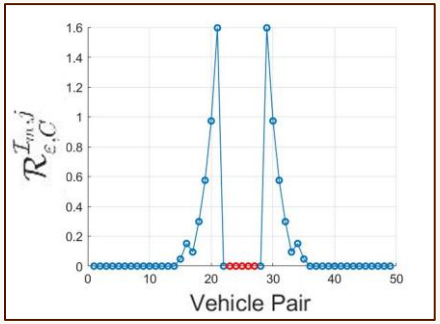




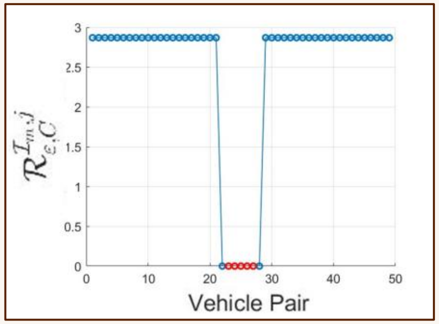
Path Graph



1-cycle Graph



5-cycle Graph

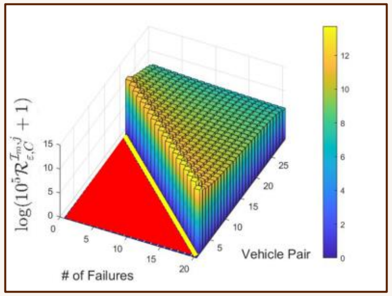


Complete Graph

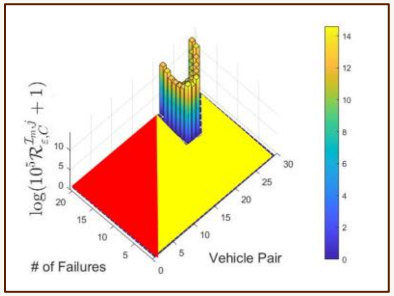
Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Characteristics of Collisions: Numbers of Existing Collisions

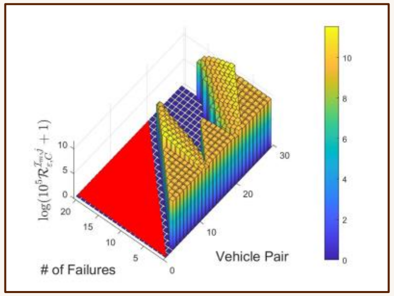
Risk profiles of a platoon with $n = 30$ vehicles, assuming pairs $\{1\}, \{1,2\}, \dots, \{1, \dots, 20\}$ have collided.



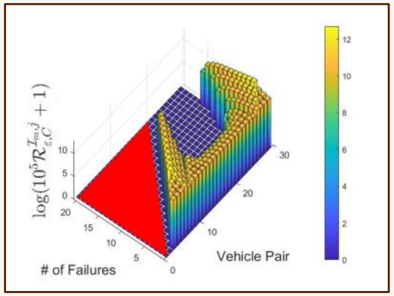
Path Graph



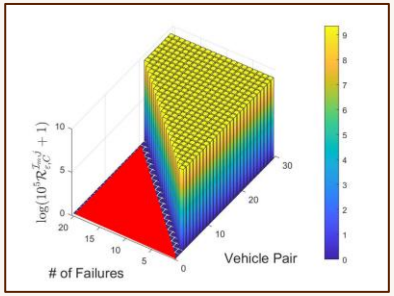
1-cycle Graph



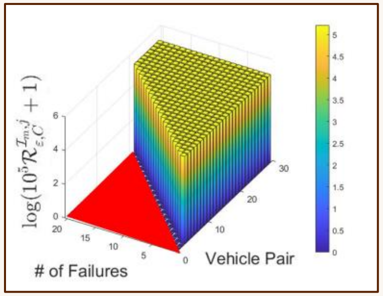
5-cycle Graph



2-cycle Graph



10-cycle Graph



Complete Graph

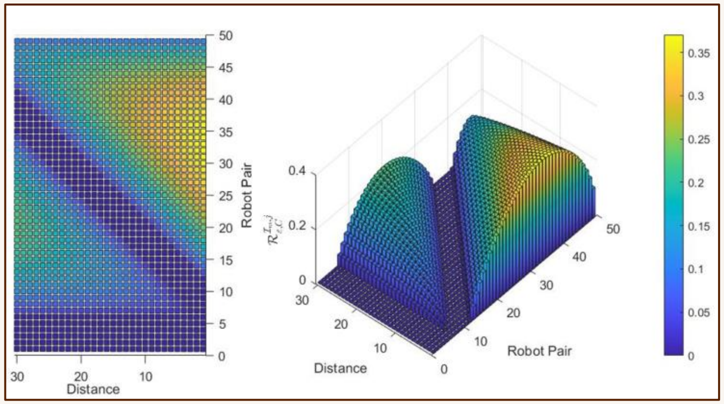
(yellow) ∞ risk

(red) pairs have collided

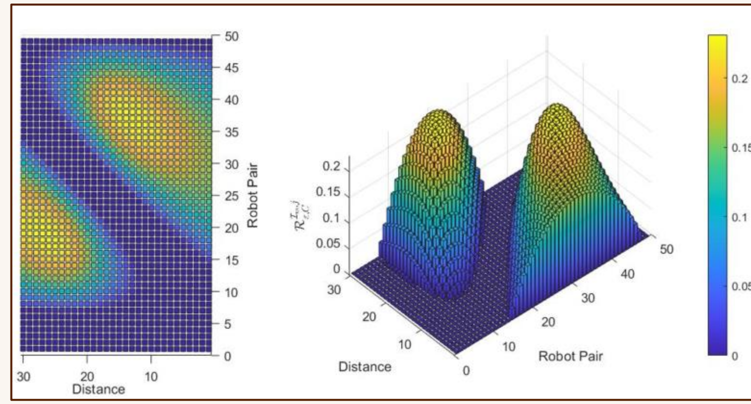
Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Characteristics of Collisions: Sparsity of Existing Collisions

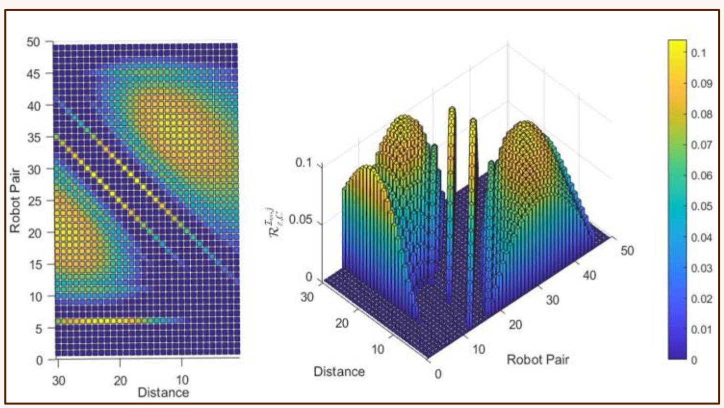
Risk profiles of a platoon with $n = 50$ vehicles, assuming pairs $\{1,2,3,4,5\}$ and $\{d+1, d+2, d+3, d+4, d+5\}$ have collided for $d = 0, \dots, 29$.



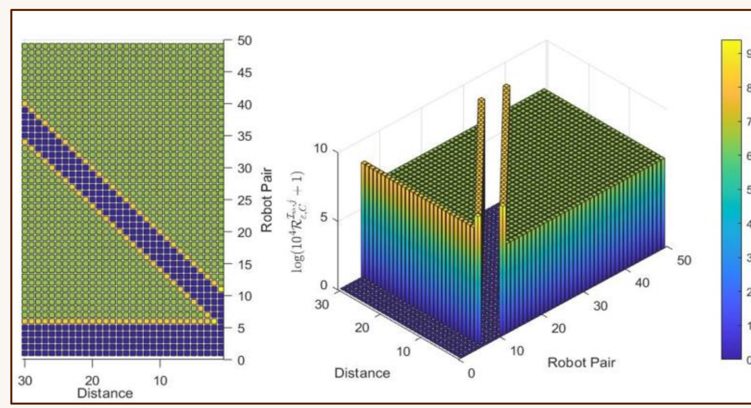
Path Graph



1-cycle Graph



5-cycle Graph



Complete Graph

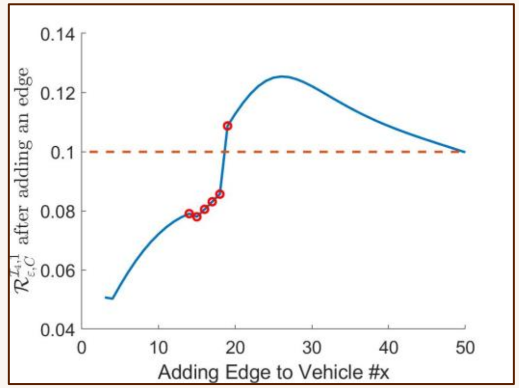
Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Characteristics of Collisions: Adding New Edges

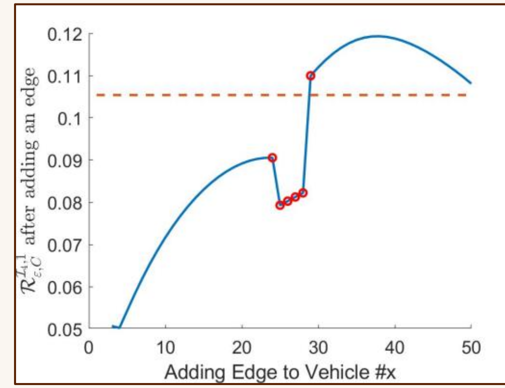
When one is allowed to alter the communication by adding an edge to the existing communication, the location of the existing failures and the added edge will both affect the risk profile.

Risk profiles of a platoon with $n = 50$ vehicles, assuming pairs {14,15,16,17,18} or {24,25,26,27,28} have collided.

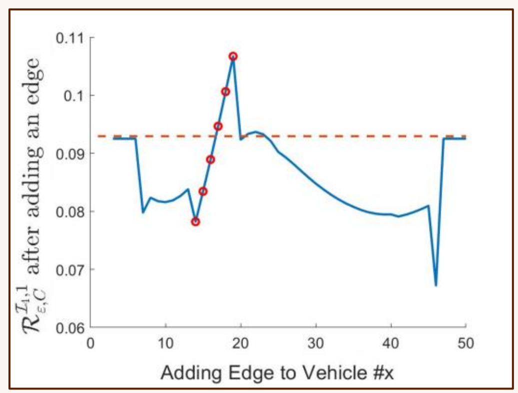
o pairs have collided



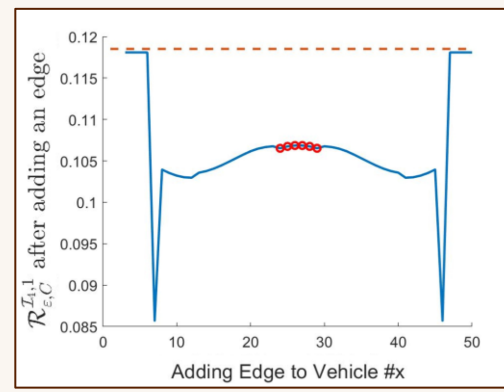
Path Graph with pair {14,15,16,17,18} have failed



Path Graph with pair {24,25,26,27,28} have failed



5-cycle Graph with pair {14,15,16,17,18} have failed



5-cycle Graph with pair {24,25,26,27,28} have failed

Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Section Overview

- Motivation
- Problem Statement
- Preliminary Result
- Risk of Inter-vehicle Collision
- Fundamental limitations and trade-offs
- Risk of Cascading Inter-vehicle Collision
- Conclusions

Conclusion

- Second-order consensus network with communication time-delay and input noise
- Steady-state statistics of the observable
- Value-at-risk framework of a single collision
- Time-delay induced fundamental limits and trade-offs
- Value-at-risk framework of cascading collisions
- The cascading risk quantifies the impact from the existing collisions on the platoon
- How changing the graph structure by adding edges will reshape the risk profile

Some Useful References

- Somarakis, Christoforos, Yaser Ghaedsharaf, and Nader Motee. "Risk of collision and detachment in vehicle platooning: Time-delay–induced limitations and tradeoffs." IEEE Transactions on automatic control 65.8 (2019): 3544-3559.

- Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Risk of Cascading Failures in Time-Delayed Vehicle Platooning." 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021.

- Liu, Guangyi, Christoforos Somarakis, and Nader Motee. "Emergence of Cascading Risk and Role of Spatial Locations of Collisions in Time-Delayed Platoon of Vehicles." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.

Risk Quantification in Networked Control Systems: Synchronous Power Network

Guangyi Liu

Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

11:25 am -12:00 pm

Section Overview

- Time-delayed Synchronous Power Networks
- Internal Stability and Measurement Statistics
- Risk of Phase Incoherence
- Fundamental limitations and trade-offs

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- Time-delayed Synchronous Power Networks
- Internal Stability and Measurement Statistics
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Problem Formulation: Synchronous Power Networks

A network of n synchronous generators connected over m transmission lines.

The i'th generator is defined through the (static) triplet (J_i, β_i, E_i) and dynamic state vector $(\theta_t^{(i)}, \omega_t^{(i)})$. Let us consider the following benchmark model

$$J_i \ddot{\theta}_t^{(i)} = -\beta_i \dot{\theta}_t^{(i)} + \sum_{j=1}^{n} E_i E_j Y_{ij} \sin(\theta_t^{(j)} - \theta_t^{(i)}) + p_i$$

for $i = 1, \dots, n$.

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Problem Formulation: Synchronous Power Networks

For fixed voltage magnitudes, admittances, and power inputs, the equilibrium point belongs to the manifold

$$\mathbb{S} = \left\{ (\theta, \omega) \in \mathbb{R}^{2n} \middle| \omega = 0 \text{ and } |\theta^{(i)} - \theta^{(j)}| < \frac{\pi}{2} \text{ with } p_i = \sum_{j=1}^{n} E_i E_j Y_{ij} \sin(\theta_t^{(i)} - \theta_t^{(j)}) \right\}$$

with $i, j = 1, \ldots, n$.

Let us consider the equilibrium point of the system as $(\theta_*, 0) \in \mathbb{S}$, and using the linearization around the equilibrium to obtain the error dynamics

$$\begin{bmatrix} d\theta_t \\ d\omega_t \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -D \end{bmatrix} \begin{bmatrix} \theta_t \\ \omega_t \end{bmatrix} dt + \begin{bmatrix} 0 \\ H \end{bmatrix} d\xi_t$$

with $D = diag\left\{\frac{\beta_1}{J_1}, \ldots, \frac{\beta_n}{J_n}\right\}, H = \eta \, diag\{J_1, \ldots, J_n\}^{-1}$ and $L = [l_{ij}]$,

$$l_{ij} = \begin{cases} J_i^{-1} E_i E_j Y_{ij} \cos(\theta_*^{(i)} - \theta_*^{(j)}) \\ -\sum_{k \neq i} l_{ik} \end{cases}$$

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Problem Formulation: Synchronous Power Networks

The state feedback control is given by

$$\begin{bmatrix} d\theta_t \\ d\omega_t \end{bmatrix} = \begin{bmatrix} 0 & I \\ -L & -D \end{bmatrix} \begin{bmatrix} \theta_t \\ \omega_t \end{bmatrix} dt + \begin{bmatrix} 0 \\ H \end{bmatrix} d\xi_t + \begin{bmatrix} 0 \\ I \end{bmatrix} u_t$$

with

$$u_t^{(i)} = -\sum_{j=1}^{n} \left[m_{ij} \left(\theta_{t-\tau}^{(j)} + \eta' d\xi_t^{(j+n)} \right) + k_{ij} \left(\omega_{t-\tau}^{(j)} + \eta' d\xi_t^{(j+2n)} \right) \right]$$

The closed-loop network can be written in a compact form

$$\begin{bmatrix} d\theta_t \\ d\omega_t \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \theta_t \\ \omega_t \end{bmatrix} dt + \boldsymbol{K} \begin{bmatrix} \theta_{t-\tau} \\ \omega_{t-\tau} \end{bmatrix} dt + \boldsymbol{H} d\xi_t$$

in which

$$\boldsymbol{A} = \begin{bmatrix} 0 & I \\ -L & -D \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} 0 & 0 \\ -J^{-1}M & -J^{-1}K \end{bmatrix}, \text{ and } \boldsymbol{H} = \begin{bmatrix} 0 & 0 & 0 \\ \eta J^{-1} & \eta' M & \eta' K \end{bmatrix} \in \mathbb{R}^{2n \times 3n}$$

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Problem Formulation: Synchronous Power Networks

In a big picture:

- A network of identical generators aim to synchronize.
- Existence of the communication time-delay, exogenous noise, and measurement noise.
- Fluctuate around the equilibrium point

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Section Overview

- Time-delayed Synchronous Power Networks
- Internal Stability and Measurement Statistics
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- Fundamental limitations and trade-offs

Simultaneous Diagonalizable Feedback Control

Assumption: The feedback gain matrices M and K are designed such that each pair out of L, M, K commutes.

An equivalent formulation: There exists a unitary matrix Q such that $Q^T U Q$ is diagonal for every $U \in \{L, M, K\}$, such that $Q^T L Q = \Lambda_L$, $Q^T K Q = \Lambda_K$, and $Q^T M Q = \Lambda_M$, where

$$\Lambda_L = \mathrm{diag}\{\lambda_1, \dots, \lambda_n\}$$
$$\Lambda_M = \mathrm{diag}\{\mu_1, \dots, \mu_n\}$$
$$\Lambda_K = \mathrm{diag}\{\kappa_1, \dots, \kappa_n\}$$

Example: Simultaneously diagonalizable structures: $L = I$, M is a graph Laplacian matrix, and K is a centering matrix

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Preliminary Result: Stability Condition

The unperturbed network will reach the steady state with non-zero time-delay $\tau > 0$ if and only if

$$\left(\tilde{d}\tau, \lambda_j\tau^2; \mu_j\tau^2, \kappa_j\tau\right) \in \bigcup_{r=0}^{3} \mathbb{W}_r$$

where

$$\mathbb{W}_0(s;k) = \left\{ s \in \mathbb{R}^2_+,\ k \in \mathbb{R}^2 \ :\ s_2 = k_1 = 0,\ \{\ |k_2| < s_1\ \} \cup \{\ k_2 > s_1,\ \sqrt{k_2^2 - s_1^2} < \text{arccot}\big(-s_1/\sqrt{k_2^2 - s_1^2}\big)\ \} \right\}$$

$$\mathbb{W}_1(s;k) = \left\{ s \in \mathbb{R}^2_+,\ k \in \mathbb{R}^2 \ :\ s_2^2 > k_1^2,\ k_2 + s_1 > 0,\ k_1 + s_2 > 0,\ k_2^2 + 2s_2 - s_1^2 \leq 2\sqrt{s_2^2 - k_1^2} \right\}$$

$$\mathbb{W}_2(s;k) = \left\{ s \in \mathbb{R}^2_+,\ k \in \mathbb{R}^2 \ :\ s_2^2 \leq k_1^2,\ k_2 + s_1 > 0,\ k_1 + s_2 > 0,\ \gamma_+(s;k) < \varphi_+(s;k) \right\}$$

$$\mathbb{W}_3(s;k) = \left\{ s \in \mathbb{R}^2_+,\ k \in \mathbb{R}^2 \ :\ s_2^2 > k_1^2,\ k_2 + s_1 > 0,\ k_1 + s_2 > 0,\ k_2^2 + 2s_2 - s_1^2 > 2\sqrt{s_2^2 - k_1^2},\ \big(\gamma_\pm(s;k), \varphi_\pm(s;k)\big) \in \mathfrak{I}_{s;k} \right\}$$

$$\begin{bmatrix} d\theta_t \\ d\omega_t \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \theta_t \\ \omega_t \end{bmatrix} dt + \boldsymbol{K} \begin{bmatrix} \theta_{t-\tau} \\ \omega_{t-\tau} \end{bmatrix} dt + \boldsymbol{H} d\xi_t$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & I \\ -L & -D \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} 0 & 0 \\ -J^{-1}M & -J^{-1}K \end{bmatrix}, \text{ and } \boldsymbol{H} = \begin{bmatrix} 0 & 0 & 0 \\ \eta J^{-1} & \eta' M & \eta' K \end{bmatrix} \in \mathbb{R}^{2n\times 3n}$$

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Preliminary Result: Observable

To quantify the risk of the phase incoherence between two generators, let us consider the observable as

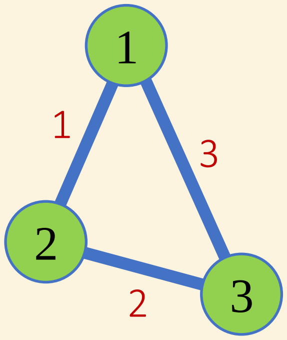
$$y_t = B_n \, \theta_t$$

where the $n\frac{n-1}{2} \times n$ complete incidence matrix B_n is given by

$$b_{ij} = \begin{cases} 1 & \text{if edge } i \text{ leaves node } j \\ -1 & \text{if edge } i \text{ enters node } j \\ 0 & \text{otherwise} \end{cases}$$

Example: In the case of 3 generators, we have

$$B_n = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$



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Preliminary Result: Steady-state Statistics

In the steady-state, the output converges, in distribution, to

$$\bar{y} \sim \mathcal{N}\left(0, \frac{1}{2\pi} B_n Q \operatorname{diag}\{\mathfrak{f}_l\}\, Q^T B_n^T\right)$$

where

$$\mathfrak{f}_l = \begin{cases} 0 & \text{if } l = 1 \\ \tau^3 \left[\frac{\eta^2}{J^2} + \eta'^2 \left(\frac{(k_1)_l^2}{\tau^4} + \frac{(k_2)_l^2}{\tau^2}\right)\right] f((s;k)_l) & \text{if } l > 1 \end{cases}$$

$(s;k)_l$ represents $(s_1, s_2; k_1, k_2)_l \coloneqq \left(\tilde{d}\,\tau, \lambda_l \tau^2; \mu_l\,\tau^2, \kappa_l \tau\right)$, and

$$f(s;k) = \int_{\mathbb{R}} \frac{dr}{2\left((s_1 k_2 - k_1) r^2 + s_2 k_1\right)\cos(r) - 2r(k_2 r^2 + s_1 k_1 - k_2 s_2)\sin(r) + r^4 + (s_1^2 + k_2^2 - 2 s_2) r^2 + s_2^2 + k_1^2}$$

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Section Overview

- Time-delayed Synchronous Power Networks
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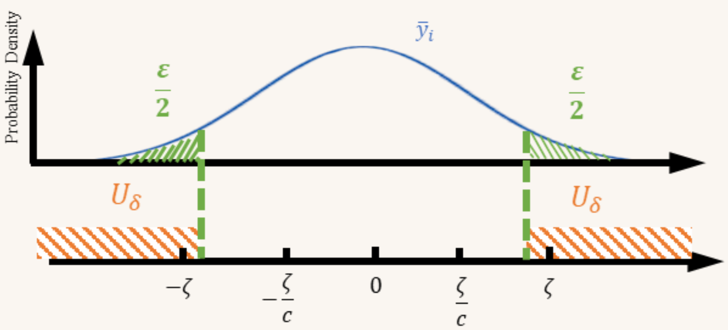
Phase Incoherence: For the observable between generators i and j, we consider the event of phase incoherence in the steady-state as

$$\{|\bar{y}^{(i,j)}| \in (\zeta, \infty)\}.$$

Level sets and Value-at-Risk Measure: A family of level sets $U_\delta = (\zeta \frac{1+\delta}{c+\delta}, \infty)$ helps to construct an alarm zone that describes how a pair of generators are dangerously close to the incoherence. The VaR measure is then given by

$$\mathcal{R}_\varepsilon := \inf\{\delta \geq 0 \mid \mathbb{P}\{\, |\bar{y}^{(i,j)}| \in U_\delta\} < \varepsilon\}.$$



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Risk of Phase Incoherence

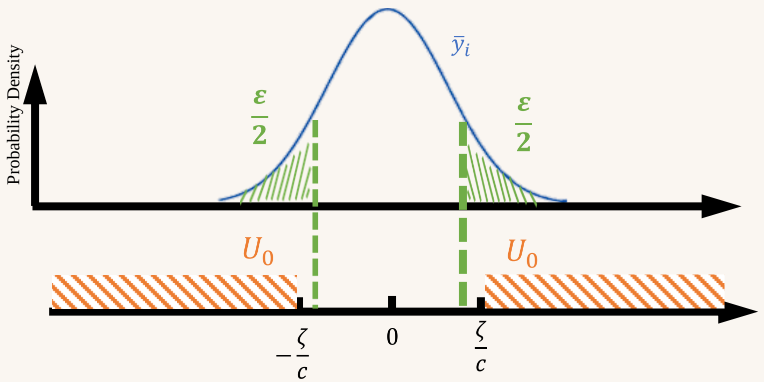
When the network reaches the steady-state, the risk of phase incoherence between generator i and j is given by

$$\mathcal{R}_\varepsilon^{(i,j)} := \begin{cases} 0 & \text{if } \sigma_{ij} \leq \dfrac{\zeta}{c\nu_\varepsilon} \\ \dfrac{\sigma_{ij}\nu_\varepsilon c - \zeta}{\zeta - \sigma_{ij}\nu_\varepsilon} & \text{if } \dfrac{\zeta}{c\nu_\varepsilon} < \sigma_{ij} < \dfrac{\zeta}{\nu_\varepsilon} \\ +\infty & \text{if } \sigma_{ij} \geq \dfrac{\zeta}{\nu_\varepsilon} \end{cases}$$

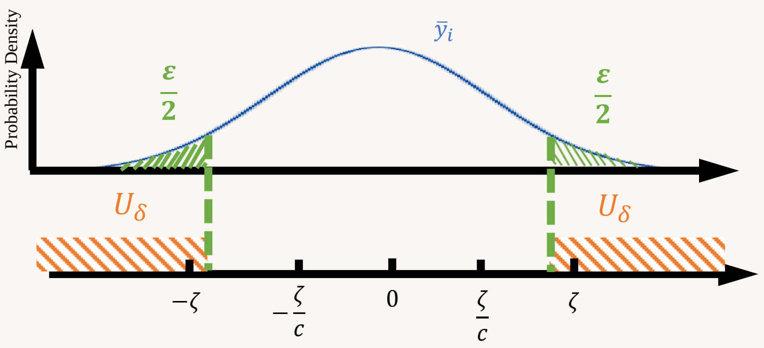
with $\sigma_{ij} = \sqrt{\dfrac{1}{2\pi}\sum_{l=2}^{n}(q_{il} - q_{jl})^2 \mathfrak{f}_l}$ and ν_ε the solution of

$$\int_{\nu_\varepsilon}^{\nu_\varepsilon} e^{-t^2/2}dt = \sqrt{2\pi}(1 - \varepsilon)$$

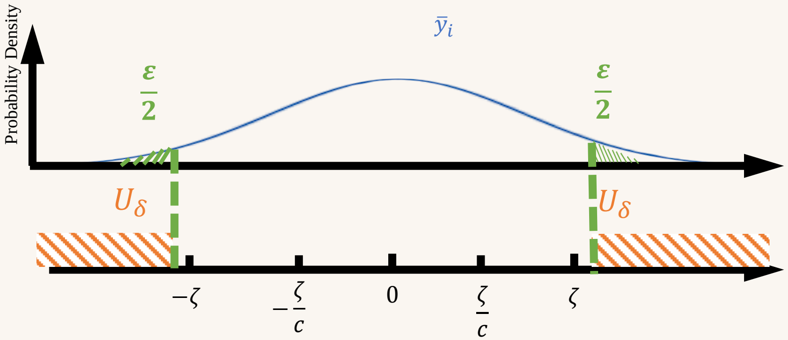
Risk of Phase Incoherence



A narrow distribution or a low confidence level



$$\frac{\sigma_{ij}\nu_\varepsilon c - \zeta}{\zeta - \sigma_{ij}\nu_\varepsilon} \quad \text{if } \frac{\zeta}{c\nu_\varepsilon} < \sigma_{ij} < \frac{\zeta}{\nu_\varepsilon}$$



A wide distribution or a high confidence level

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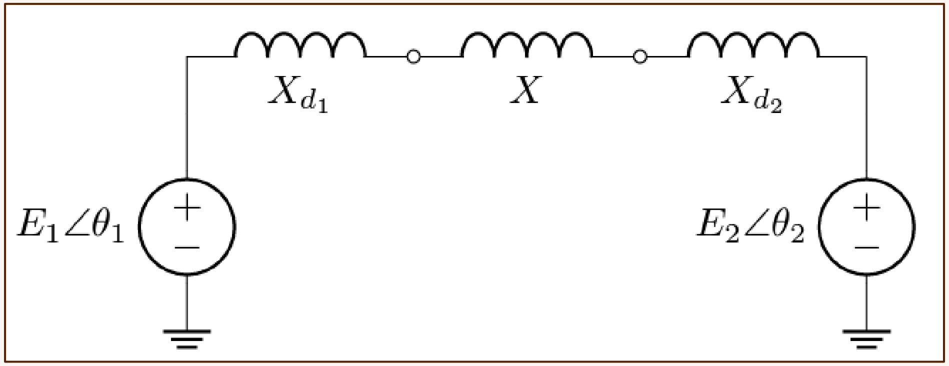
Risk of Phase Incoherence: Two-Machine System



$$2\,\ddot{\theta}_t^{(1)} = -0.15\,\dot{\theta}_t^{(1)} + 1.584\,(\theta_t^{(2)} - \theta_t^{(1)}) + \mathrm{distrb}_1$$
$$2\,\ddot{\theta}_t^{(2)} = -0.15\,\dot{\theta}_t^{(2)} + 1.584\,(\theta_t^{(1)} - \theta_t^{(2)}) + \mathrm{distrb}_2$$

Consider the phase difference as $\theta_t = \theta_t^{(1)} - \theta_t^{(2)}$, and both generators use the uniform feedback control gain, then

$$2\,\ddot{\theta}_t = -0.15\,\dot{\theta}_t - 3.168\,\theta_t - \kappa\,\dot{\theta}_{t-\tau} - \mu\,\theta_{t-\mu} + \mathrm{distrb}$$

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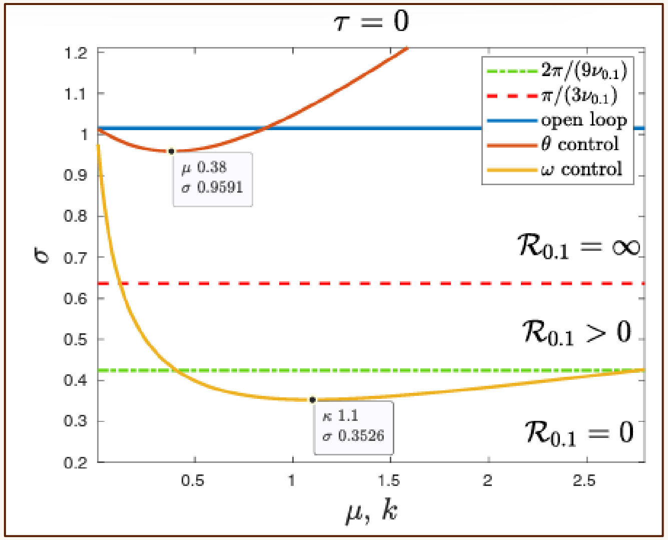
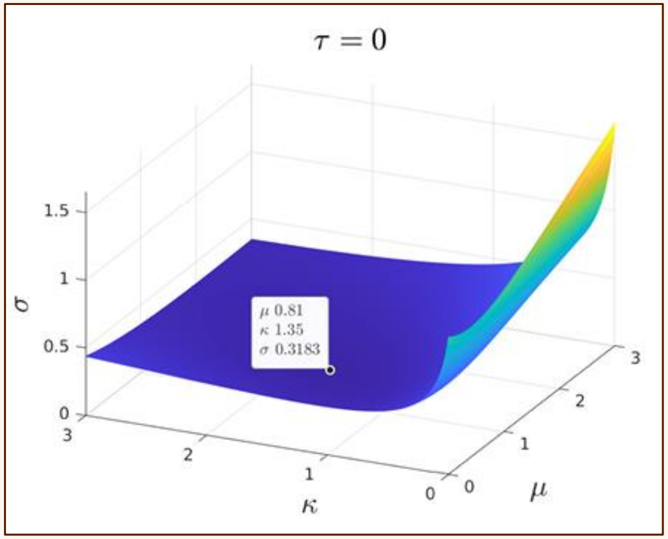
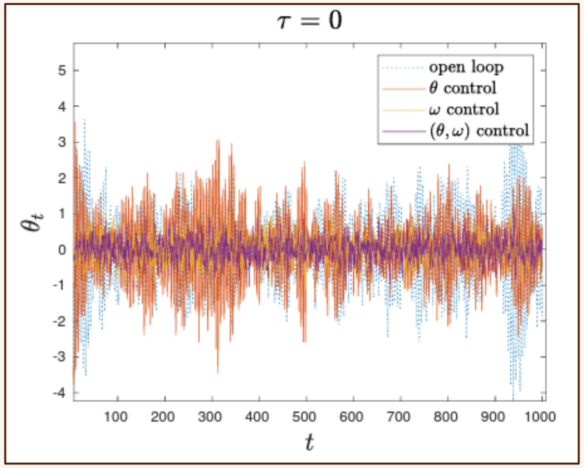
Risk of Phase Incoherence: Two-Machine System



Uniform μ and κ



Separate μ and κ



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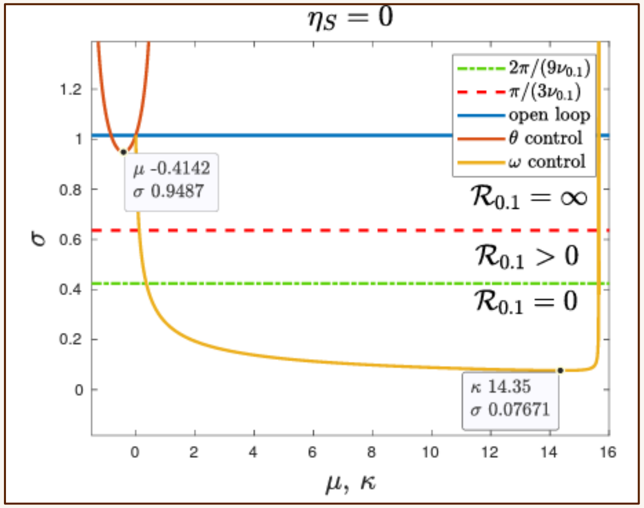
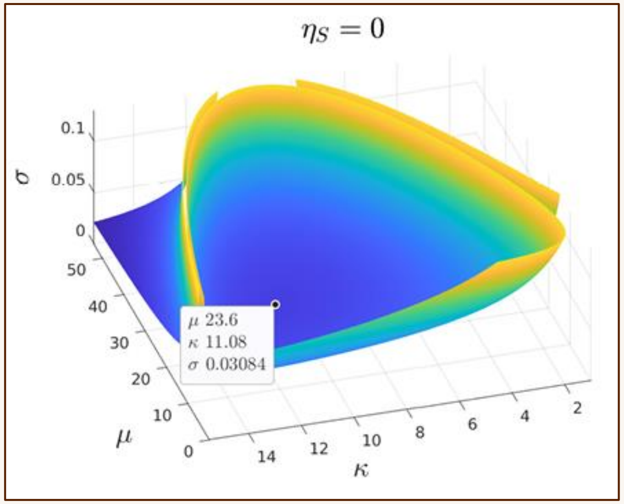
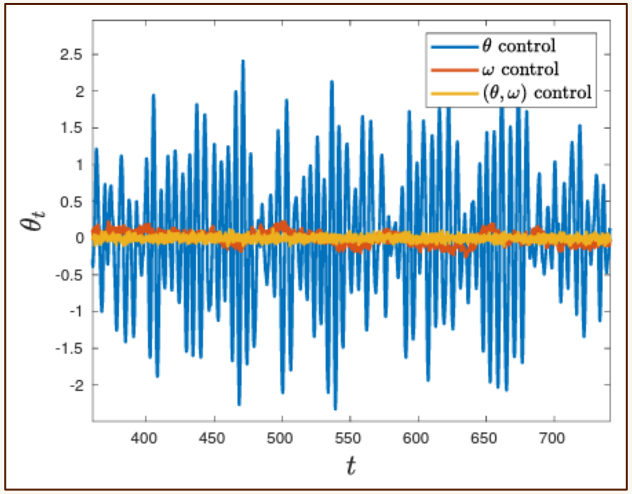
Risk of Phase Incoherence: Two-Machine System



Uniform μ and κ



Separate μ and κ



Somarakis, Christoforos, Guangyi Liu, and Nader Motee. "Risk of Phase Incoherence in Wide Area Control of Synchronous Power Networks." *submitted to IEEE-TAC*

Section Overview

- Time-delayed Synchronous Power Networks
- Internal Stability and Measurement Statistics
- Risk of Phase Incoherence
- Fundamental limitations and trade-offs

Time-delay induced fundamental limits

Due to the existence of the time-delay and the stability constraint, the variance of the phase difference of any two generators is lower bounded by

$$\sigma_{ij} \geq \sigma_* := \frac{\tau^{3/2}\eta}{J\sqrt{2\pi}} \min_{(i \neq j)} \sqrt{\sum_{l=2}^{n} (q_{il} - q_{jl})^2 \underline{f_l}}$$

with

$$\underline{f_l} := \min_{(s_l;k) \in \bigcup_{r=1}^{3} \mathbb{W}_r} f((s_l;k)).$$

Independent to the feedback control design !!!

Somarakis, Christoforos, Guangyi Liu, and Nader Motee. "Risk of Phase Incoherence in Wide Area Control of Synchronous Power Networks." *submitted to IEEE-TAC*

Trade-offs

For a particular type of the state feedback controller with

$$M = \mu L \text{ and } K = \kappa L,$$

there exists a best achievable lower bound on the product of the risk and the power network connectivity.

Given systemic set parameters ζ, c, and the acceptance level $\varepsilon \in (0,1)$, there exists a common limit for the product of the systemic risk and the effective resistance

$$\mathcal{R}_\varepsilon \cdot \sqrt{\Xi_K + \Xi_M} \geq \Omega$$

where Ω is a universal constant depending on the grid properties, time-delay, and uncertainty constants η, η'.

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Conclusion

- Synchronous power network with communication time-delay, input noise, and exogeneous noise
- Stability condition for the phase consensus
- Steady-state statistics of the observable
- Value-at-risk framework of phase incoherence
- Time-delay induced fundamental limits and trade-offs

Somarakis, Christoforos, Guangyi Liu, and Nader Motee. "Risk of Phase Incoherence in Wide Area Control of Synchronous Power Networks." *submitted to IEEE-TAC*

Risk Quantification in Networked Control Systems: Robotics Perception Risk

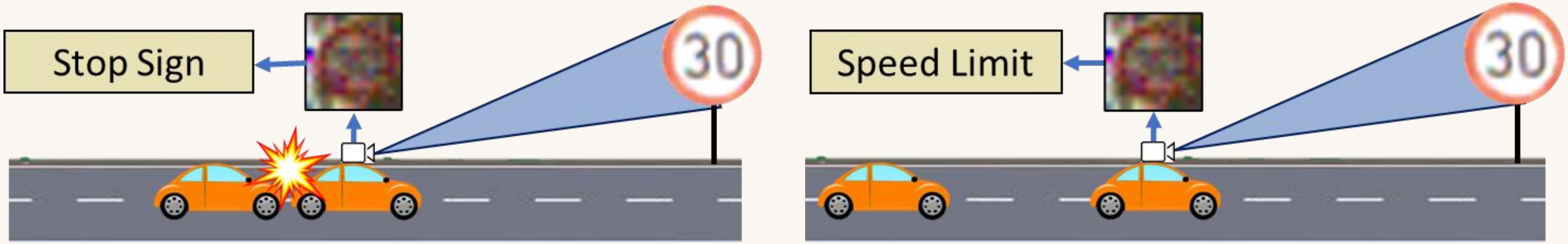
Guangyi Liu
Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA, 18015

12:00 am -12:20 pm

Section Overview

- Motivation
- Problem Formulation
- Data-driven Statistics Estimation
- Cost Metric and the Construction of AV@R
- Case Study

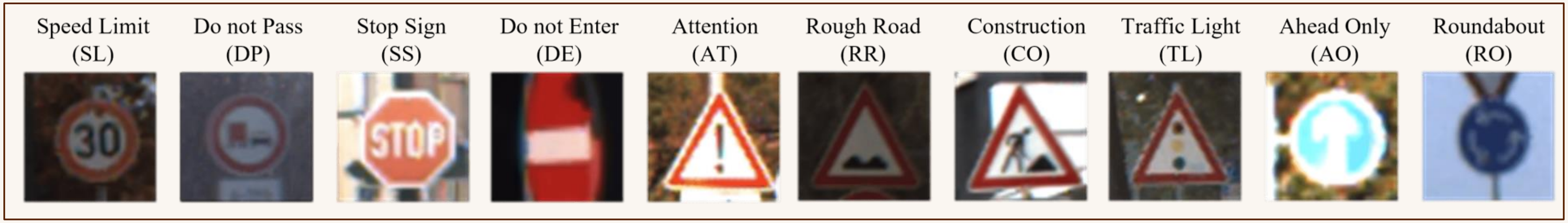
Motivation



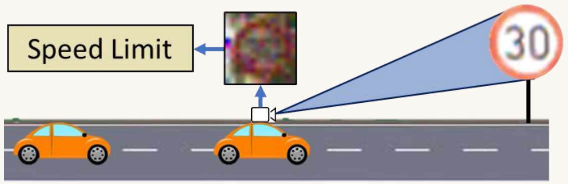
Motivations:

- The environment (visual input) is always noisy.
- The autonomous driving vehicle is prone to make unsafe decisions with noisy input.
- Such unsafe decisions may result in a cascade of accidents or violation of traffic rules.

Problem Formulation

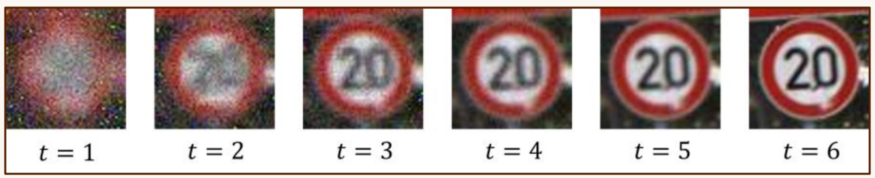


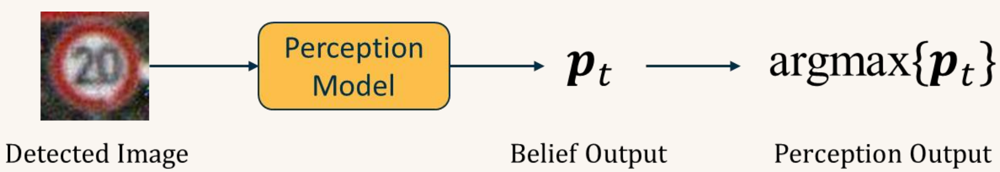
- Autonomous driving vehicle equipped with onboard perception unit to classify the detected traffic sign.
- The detected image of the traffic sign suffers from a time-varying resolution and additive Gaussian noise.
- Evaluating the risk of misperceiving the traffic sign and find the safest decision (action).



Liu, G., Kamale, D., Vasile, C. I., & Motee, N. (2023). Symbolic Perception Risk in Autonomous Driving. 2023 American Control Conference (ACC 2023)

Problem Formulation





- A simple VGG-19 model with a SoftMax layer, trained with original images from the dataset.
- For each detected image, the perception unit generates a belief output $p_t \in \mathbb{R}^{10}$.
- The belief output p_t lies with in a $\mathbb{R}^9$ simplex.

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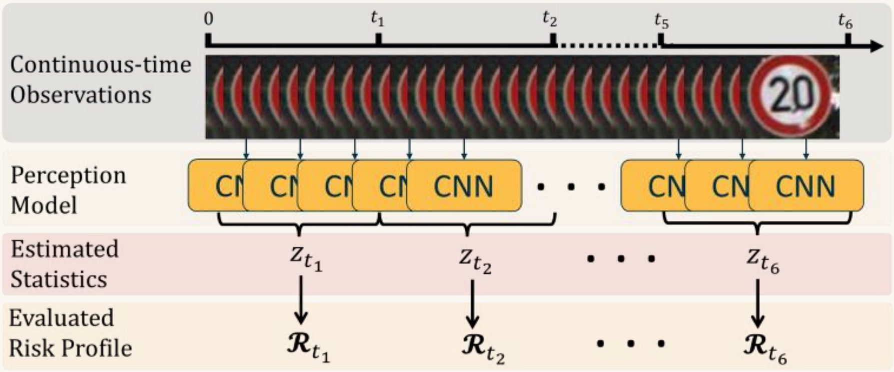
Data-driven Statistics Estimation

Unlike any of the previous case, we can not solve a system equation to obtain the statistics of the output, i.e., p_t.

We will consider the data-driven approach to obtain an accurate estimation of the output statics instead.



- We assume that the statistics of p_t do not change drastically in any sufficiently short time interval.
- For each short time interval, the vehicle is able to collect sufficient amount of belief output p_t's.

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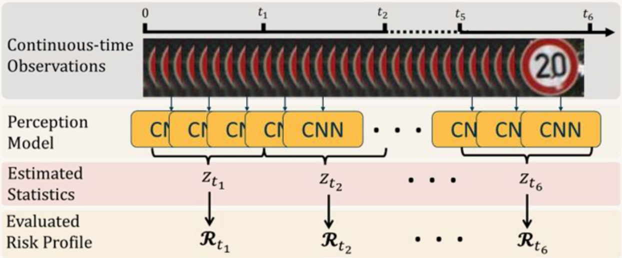
Data-driven Statistics Estimation

Considering the fact that p_t lie within the simplex, it is intuitive to consider estimating its statistics by the Dirichlet distribution, for which its density function is given by



$$f_{\mathcal{D}}(z_1, \dots, z_m; \alpha_1, \dots, \alpha_m) = \frac{\Gamma(\sum_{i=1}^{m} \alpha_i)}{\prod_{i=1}^{m} \Gamma(\alpha_i)} \prod_{i=1}^{m} z_i^{\alpha_i - 1},$$

and it enjoys the following property

$$\sum_{i \in \mathcal{M}} z_i = 1, \text{ and } z_i \geq 0$$

The estimated value of $\boldsymbol{\alpha}_t$ can be updated as follow given the set of belief outputs, i.e.,

$$\Psi(\alpha_{t,i}^{new}) = \Psi\left(\sum_{j=1}^{m} \alpha_{t,j}^{old}\right) + \frac{1}{q} \sum_{t' \in \mathcal{T}_t^{\tau}} \log p_{t',i}$$

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Cost of Traffic Sign Misperception

Misperceiving traffic signs often leads to poor decisions from autonomous vehicles, which are primarily associated with high potential costs in real-world driving scenarios.

Simply interpreting the belief output as “correct” or “wrong” does not provide adequate information for safe autonomous driving since the high-level actions associated with each traffic sign do not yield the same potential cost.

Sign	SL	DP	SS	DE	AT	RR	CO	TL	AO	RO
SL	0	174	103	103	123	123	121	103	121	120
DP	117	0	105	105	117	117	119	105	97	113
SS	135	109	0	96	110	110	110	96	135	135
DE	117	117	99.5	0	117	500	117	117	117	117
AT	71	111.5	102	92	0	50	0	102	51	137.5
RR	144.5	168	82	82	50	0	50	140	168	258
CO	102	41.5	82	82	30	0	0	41	83	173
TL	97	97	77.5	77.5	39	73	73	0	73	163
AO	91	91	86.5	86.5	45.5	45.5	45.5	91	0	182
RO	83	83	165	165	41.5	41.5	41.5	63	200	0

TABLE I: The cost metric for traffic sign misperception (unit: €1000).

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Connecting the Perception Output to Cost

In order to quantify the risk of misperceiving on traffic sign into another, we should construct a new random variable r that represents the cost associated with the perception output z_t,

$$r_i(z_t) = C_{ji} \text{ if } z_t \in V_j$$

Sign	SL	DP	SS	DE	AT	RR	CO	TL	AO	RO
SL	0	174	103	103	123	123	121	103	121	120
DP	117	0	105	105	117	117	119	105	97	113
SS	135	109	0	96	110	110	110	96	135	135
DE	117	117	99.5	0	117	500	117	117	117	117
AT	71	111.5	102	92	0	50	0	102	51	137.5
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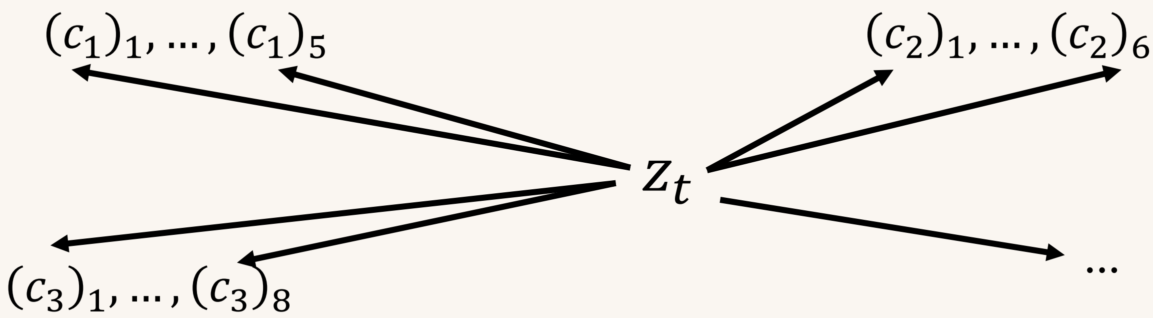
TABLE I: The cost metric for traffic sign misperception (unit: €1000).

Are we good to go?

There might be a case when different traffic sign will yield the same cost, and the cost are not ordered yet. For each label i, we sort the unique cost values as

$$\max_{j \in \mathcal{M}} C_{ji} = (c_i)_1 > \ldots > (c_i)_{m'_i} = \min_{j \in \mathcal{M}} C_{ji}$$

Probability of Misperception



For each element of ordered cost vector c_i, the probability of $\mathbb{P}\{r_i(z_t) = (c_i)_j\}$ is given by

$$\mathbb{P}\{r_i(z_t) = (c_i)_j\} = \hat{p}_{t,j} = \sum_{k | C_{k,i} = (c_i)_j} \mathbb{P}\{z_t \in V_k\}$$

where

$$\mathbb{P}\{z_t \in V_k\} = \int_0^\infty \prod_{i \neq k} \left(\frac{\gamma(\alpha_{t,i}, x)}{\Gamma_{\alpha_{t,i}}} \right) \frac{x^{\alpha_{t,k} - 1} \exp(-x)}{\Gamma(\alpha_{t,k})} \, dx$$

Risk of Misperception

During each small time interval, given the estimated belief output z_t, the risk of misperception with the i'th label is given by

$$\mathcal{R}^{\varepsilon}_{t,i} = \frac{1}{\varepsilon}\left(\sum_{j=1}^{v}(c_i)_j\, \hat{p}_{t,j} + (c_i)_{v+1}\Big(\varepsilon - \sum_{j=1}^{v}\hat{p}_{t,j}\Big)\right),$$

where the integer value v is given by

$$v = \sup_{v \leq m_i'} \sum_{j=1}^{v}\hat{p}_{t,j} \leq \varepsilon$$

Key observations:

- AV@R for discrete random variable
- Splitting a probability atom

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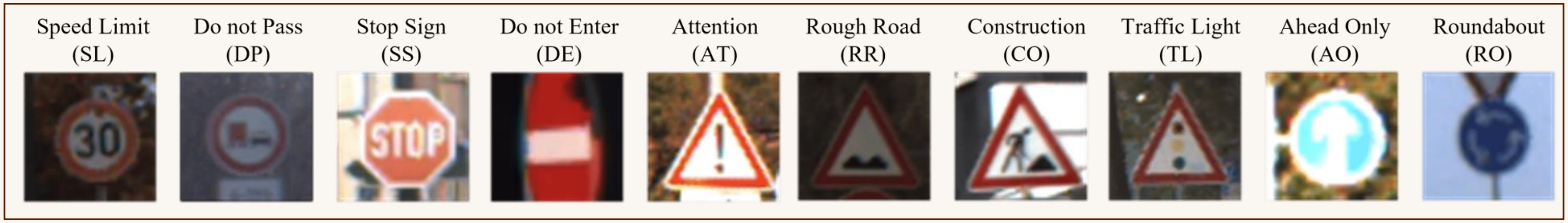
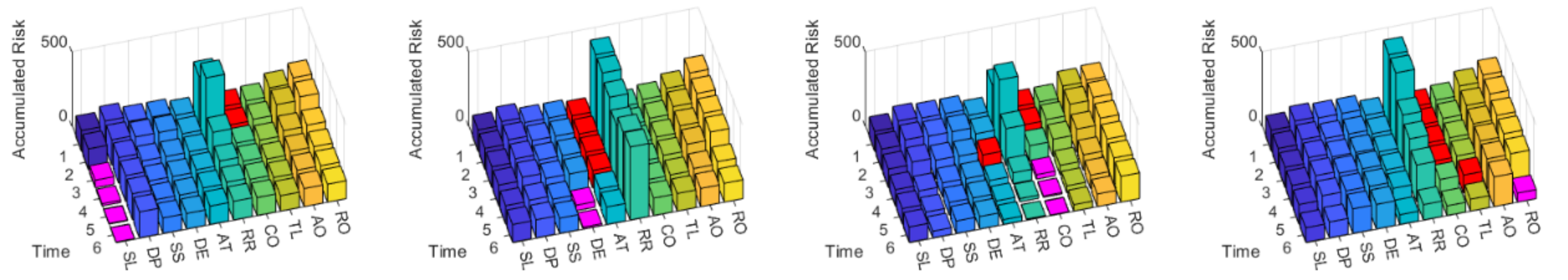
Case Study





(a) True label: Speed-limit (SL) $\varphi(SL)$ =*Slow_down*

(b) True label: Do not enter (DE) $\varphi(DE)$ =*Slow_down & Change_direction*

(c) True label: Construction (CO) $\varphi(CO)$ =*Go_slow & Caution*

(d) True label: Roundabout (RO) $\varphi(RO)$ =*Follow_directions*

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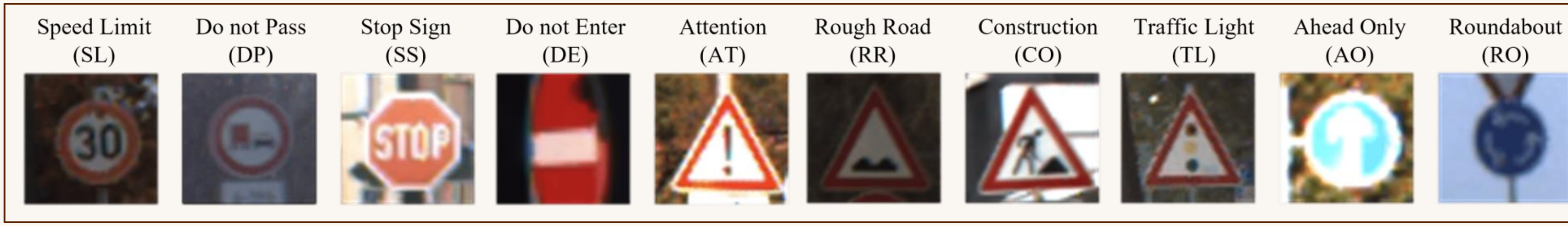
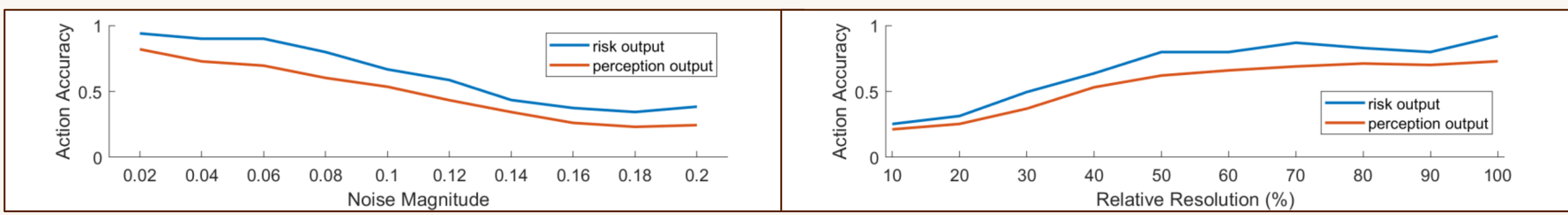
Case Study





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Conclusion

- Autonomous driving vehicle that detect and classify the traffic sign
- The detected traffic sign suffers from the time-varying noise and resolution change
- Estimation of the statistics of the belief output
- Construction of the AV@R and the cost metric
- Evaluating the risk in terms of cost, but not the perception output

Contact Information

Guangyi Liu	gliu@lehigh.edu
Vivek Pandey	vkp219@lehigh.edu
Nader Motee	motee@lehigh.edu

Workshop Website

Building C

113 Research Drive

Mountaintop Campus

Bethlehem, PA 18015

Lehigh University AIRLab