Matching Code-on-Graph with Network-on-Graph: Adaptive Network Coding for Wireless Relay Networks

Xingkai Bao, and Jing Li (Tiffany)
Department of Electrical and Computer Engineering,
Lehigh University, Bethlehem, PA 18015
Email: { xib3, jingli}@ece.lehigh.edu

Abstract

We consider user cooperation in a wireless relay network that comprises a collection of transmitters sending data to a common receiver through inherently unreliable and constantly changing channels. Exploiting the recently developed technology of network coding, we propose a new framework, termed *adaptive network coded cooperation* (ANCC), as a practical, de-centralized, adaptive and efficient cooperative strategy for large-scale wireless networks. The key idea is to match network-on-graph, i.e. the instantaneous network topology described in graph, with the well-known class of codes-on-graph, i.e. low-density parity-check (LDPC) codes and LDPC-like codes, to combat the lossy nature of wireless links and to adapt to the changing network topology. We demonstrate, through several ensembles of low-density generator matrix (LDGM) codes and lower-triangular LDPC codes, that the proposed framework can considerably increase the cooperation level and reduce the outage rate. Huge gains of some 20-40 dBs are achieved over the conventional *repetition* schemes!

Index Terms

User cooperation, relay networks, network coding, code-on-graph, low-density parity-check (LDPC) codes, low-density generator-matrix (LDGM) codes

I. Introduction

User cooperation, or the relay channel problem, was first discussed by van der Meulen in [1]. Substantial advances in the theory and the basic coding strategies were made by Cover and El Gamal [2]. Recently, it emerged as a spatial diversity technique in fading environments that can effectively reduce the outage rate and increase the sum rate of the users [3]-[8].

The basic relay model consists of three terminals: a source S, a relay R and a destination D. In the case of symmetric cooperation, the source and the relay may alternate roles to achieve a rate and load balance. A number of excellent relaying strategies have been developed for the basic model, including, for example, *repetition*, *coded cooperation* [6] and *space-time cooperation* [4], *coded space-time cooperation* [8] and *double coded space-time cooperation* [5]. Extensive analysis and simulations have revealed substantial gains in error rate performance, power efficiency, outage probability and throughput.

Encouraged by the benefits promised by user cooperation, recent research work starts to probe into a more fruitful area that extends the three-terminal system to a many-terminal *relay network*. User cooperation in the (wireless) network context is particularly beneficial: while an individual channel operating alone may be useless due to the channel condition, combined together a set of channels may become useful again; likewise, a single node performing alone may not be able to accomplish anything, but collaboratively a number of nodes may achieve a big thing (and lead to considerable resource saving).

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The model of interest here comprises a collection of users (or sources) sending wireless data to a common destination with the assists from a collection of relays. The relays may either be special nodes deployed for the sole purpose of forwarding packets, or ordinary users that happen to be idle for the moment. Our primary interest concerns *practical* cooperative strategies that can pull together all the available dimensions of channels and network to enable efficient and trust-worthy network communication. In particular, the question we ask is: how to make the nodes cooperate efficiently in a large wireless network given the constraints of inherently unreliable channels and possibly changing topologies?

A natural and straightforward solution is to extend the existing cooperative strategies developed for the three-terminal model, such as *repetition*, *coded cooperation*, and *space-time cooperation*, to the network context [4]. However, the schemes developed for the basic system do not necessarily scale well, and may be expensive, inefficient or even wasteful to operate as the network size increases. For example, extending *repetition* to N terminals costs a large overhead of bandwidth and extending *space-time cooperation* to N terminals requires stringent inter-user synchronization among many relay nodes, which is technically challenging [4].

What we propose here is a new and different approach. From the communication perspective, what has been exploited in user cooperation is the well-known *cooperative diversity*, that is, spatial diversity provided by the virtual antenna arrays and the independent user-destination channels. From the network perspective, however, message relaying is essentially a means of routing, which can increase the dynamic range of a wireless user (through multi-hop relaying) and/or improve the end-to-end transmission reliability (by providing redundant copies of the same packet).

In the context of routing, one recently developed technology, formerly known as *network coding*, immediately becomes relevant [9]-[10]. Traditional replicate-and-forward routing techniques fail to achieve the communication capacity in network settings. By allowing intermediate relaying nodes to perform intelligent packet combining in the symbol level, network coding provides new capabilities to conventional routing, and opens the possibility for accomplishing optimal performance in (lossless) networks [9]-[10] Hence, networking coding can also be considered as generalization of routing or “intelligent routing.”

While the prevailing assumption in the network coding literature has been the *lossless* network where transmission though each channel is upper limited by the channel capacity but is otherwise noiseless, here we consider wireless relay networks that are formed from quasi-static Rayleigh fading channels. Although lossy, the nature of the wireless media actually facilitates network coding since broadcast is achieved without additional cost. We demonstrate, through the design of practical cooperative schemes, that the technologies of network coding can be exploited together with user cooperation (and opportunistic forwarding) to enable intelligent routing and efficient communication in a (large) wireless relay network. The framework we propose, termed *adaptive network coded cooperation* (ANCC), is distributed and adaptive in nature. The key idea is to couple *network-on-graph*, i.e. the instantaneous network topology described in graphs, with the well-known class of *codes-on-graph*, i.e. low-density parity check (LDPC) codes and LDPC-like codes, to construct efficient linear network codes that can account for the changing and lossy nature of wireless networks. The feasibility and efficiency of the proposed scheme is demonstrated through the examples of several ensembles of low-density generator-matrix (LDGM) network codes [11] and lower-triangular LDPC (LT-LDPC) network codes.

The remainder of the paper is organized as follows. Section II presents the central idea of exploiting network coding in wireless user cooperation and the concept of matching code graphs with network graphs. Section III and Section IV discuss in detail the proposed ANCC scheme using low-degree LDGM ensembles and LT-LDPC ensembles, respectively.Finally, Section V provides concluding remarks.

II. The Basic Idea of Adaptive Network Coded Cooperation

A. Network Coding for Wireless Relay Systems

We start by demonstrating the relevance of network coding to wireless user cooperation. Consider a simple but generalizable network model where two sources send wireless data, denoted as $\mathbf{a}$ and $\mathbf{b}$ respectively, to a common destination with the assist from other users. The simplest way to attain cooperative diversity is, as depicted in Figure 1(A), to "reproduce" the three-terminal model by coupling each source with each relay, such that each data symbol/packet enjoys a diversity order of 2. A more clever way, however, is to let one relay simultaneously help the two users by relaying the coded symbol/packet, $\mathbf{a}\oplus\mathbf{b}$, instead of individual source symbol $\mathbf{a}$ or $\mathbf{b}$; see Figure 1(B). It is easy to see that each source continues to enjoys a diversity of two, and retrieving any two of the three transmissions at the destination will losslessly recover both sources. Analysis using the error rate of Rayleigh fading channels ($\epsilon\approx\frac{1}{4\mathrm{SNR}}$ for large SNRs) further reveals that the simple coding strategy in the latter achieves a lower outage probability for a smaller bandwidth (while the total energy being the same).

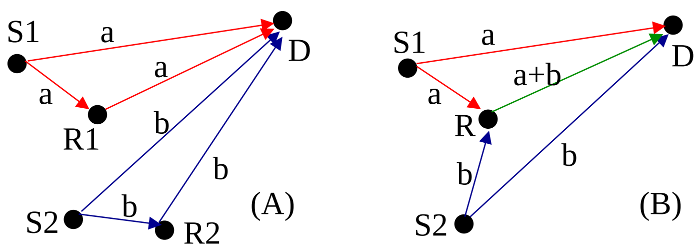

Fig. 1. A simple relay network model. (A) A conventional relay strategy. (B) A network coded relay strategy.

From the network coding perspective, the gain of the latter strategy results from the efficiency of a parity check code, $[\mathbf{a}, \mathbf{b}, \mathbf{a}\oplus\mathbf{b}]$, over a repetition code, $[\mathbf{a}, \mathbf{b}, \mathbf{a}, \mathbf{b}]$. When there are four transmit sources, the $(7, 4)$ Hamming code or the $(8, 4)$ extended Hamming code are readily applicable[1] For larger networks, more sophisticated (systematic) linear channel codes and especially powerful erasure codes can be exploited. They include, for example, the *fixed-rate* class of BCH codes and LDPC codes and the *rateless* class of Luby-Transform (LT) codes [12] and Raptor codes [13]. The application of these codes in networks is straightforward, except the practicality that real (user-user) channels are inherently unreliable. What if a relay fails to demodulate/decode a source symbol that is needed to perform its pre-specified network coding function?

In the example shown in Figure 1(B), when the relay fails to get a correct copy of $\mathbf{a}$ from User 1, it can simply forward data $\mathbf{b}$ for User 2. The parity check network code will then reduce to a repetition network code $(\mathbf{b}, \mathbf{b})$. Now what about a general case? How in general does a relay node adapt to the changing link condition and network topology *without centralized control*?

[1]The $(7, 4)$ Hamming code takes the form of $[\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{a}\oplus\mathbf{b}\oplus\mathbf{c}, \mathbf{a}\oplus\mathbf{b}\oplus\mathbf{d}, \mathbf{a}\oplus\mathbf{c}\oplus\mathbf{d}]$. The $(8, 4)$ extended Hamming code takes the form of $[\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{a}\oplus\mathbf{b}\oplus\mathbf{c}, \mathbf{a}\oplus\mathbf{b}\oplus\mathbf{d}, \mathbf{a}\oplus\mathbf{c}\oplus\mathbf{d}, \mathbf{b}\oplus\mathbf{c}\oplus\mathbf{d}]$.

B. Matching Code-on-Graph with Network-on-Graph

One possible solution to the challenge of inter-user outage is to design a family of "nested" codes such that one is the degenerated case of the other. However, this approach is cumbersome as well as limited in the degree of flexibility and adaptivity. In this paper, we propose to generate distributed linear codes on-the-fly to *match* to the instantaneous network topology.

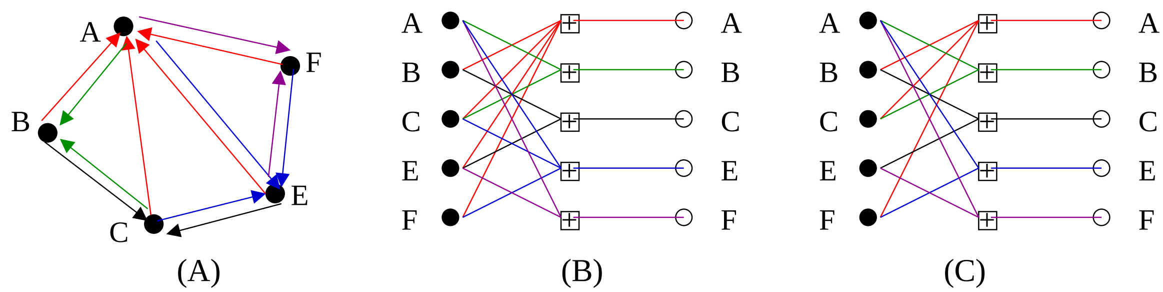

Fig. 2. Transforming a network graph to a bipartite code graph. (A) A network graph used to describe the (instantaneous) network topology. (B) A bipartite code graph derived from the network graph. (C) A thinned bipartite code graph for more efficient message-passing decoding. (● - source symbol/packet; ○ - relay symbol/packet; ⊞ - check operation.)

Consider a relay network where 5 users A, B, C, E, F communicate to a common destination D. Initially, each user broadcasts its wireless data in orthogonal channels (orthogonal either in time, frequency or spread code). Assume for the time being, the qualities of the inter-user channels are as shown in Figure 2(A) (the destination D is not shown), where a directed edge indicates a successful transmission. Without much manipulation, we can transform this network graph to a bipartite code graph in Figure 2(B). Following the convention of code graph, let us use boxes to represent *check-nodes* and (black/white) circles to represent *bit-nodes*. For N transmitters, there will be N black circles representing the source symbols originated by these nodes, N boxes representing the network coding operations performed at these nodes, and N white circles representing the relay symbols forwarded by these nodes. The transformation of the network graph to the bipartite code graph is done in two simple steps: (1) connect a source variable-node X (i.e. white circle) and a check-node Y (box) in the code graph with an edge if there is a directed link from user X to user Y in the network graph, and (2) connect each check-node (box) with its corresponding relay variable-node (black circle) in the code graph. As shown in Figure 1(B), a $(10, 5)$ systematic LDPC-like network code thus results, where each user relays the check sum of its correctly-decoded symbols. We see that such a graph-based network code is constructed on-the-fly and matches well with the instantaneous network topology. A small bit-map field will be included in each relay packet, so that the destination knows how the checks are formed and can correspondingly replicate the code graph and perform message-passing (or belief-propagation) decoding. This requires an adaptive decoder architecture, which can be implemented, for example, using software-defined radio (SDR).

To make the message-passing algorithm effective, instead of performing the check sum on *all* the decoded symbols, each node can (randomly) pick only *a few* symbols, thus "thinning" the code graph and eliminating the chances for short cycles. For example, in Figure 2(A), user A may de-select the source symbol from user E when performing the coding operation. Likewise, user E may leave out the source symbol from user C. The new code graph, which now happens to be free of length-4 cycles, is shown in Figure 2(C).

Additionally, depending on the quality of user-destination channels and/or power supplies, not all users need to participate in the relay process. Research work on (the downlink of) the cellular type of networks has revealed considerable gains enabled by *user diversity*. With the help of certain coordination or control packets, it is possible to exploit user diversity, opportunistic relaying, and resource management together with the adaptive network coding strategy in user cooperation. A joint design and optimization of these useful technologies will further improve the efficiency of the proposed *adaptive network coded cooperation*.

III. Low-Density Generator-Matrix Codes for Large Relay Networks

A. Adaptive LDGM Network Codes

To fully explore the idea we just proposed, let us examine several ensembles of LDPC/LDPC-like codes as candidate pools for on-the-fly network codes. We start with low-density generator-matrix (LDGM) codes.

LDGM codes are a special class of linear-time encodable LDPC codes. The parity check matrix of a systematic LDGM code consists of two parts, a sparse (and random) matrix P on the left and a unit matrix I on the right, i.e. $H = [P, I]$; see Figure 3. Due to its special structure, the generator matrix of LDGM code is also sparse (the generator matrix of an LDPC code may be dense). An LDGM code is attractive for its small storage (memory) requirement and the low encoding complexity, which scales linearly with the block size N: $O(N)$. However, since the bits corresponding to the identity part of H lack adequate protection (each bit participates in only one check), LDGM codes have a shortcoming of exhibiting high error floors [11]. Therefore, caution should be taken in utilizing an LDGM code such that the error floor appears low enough to meet the application requirement. This will be further discussed in Subsection III-B.

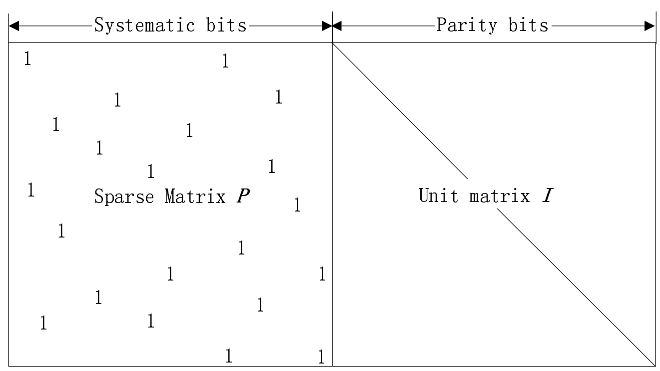

Fig. 3. The parity check matrix of an LDGM code.

Let us consider ensembles of rate 1/2 regular LDGM codes. An LDGM code is called *regular*, if all the columns in P have the same number of "1"s and all the rows in P have (approximately) the same number of "1"s. The constant column weight is termed "degree" and can be used to specify an LDGM ensemble. The basic idea of using ensembles of LDGM codes as topology-adaptive network codes is straightforward. In the broadcast phase, depending on the quality of the user-user channels, each terminal in the network receives part or all of others' data symbols which form a network graph. As discussed previously, this instantaneous network graph, or its thinned version, can transform to a bipartite code graph. To increase the girth of the resulting code graph and simplify the decoding process in the destination, in the relay phase, each terminal randomly selects a small, fixed number (say D) of data symbols from its set of received symbols, and transmits their binary check sum to the destination. To put network coding in perspective, we note that the source symbols transmitted in the broadcast phase and the check-sum symbols transmitted in the relay phase form respectively the systematic and the parity part of a degree-D rate 1/2 LDGM codeword (see Figure 3). They can therefore be decoded using the message-passing algorithm at the destination.

Clearly, with each round of broadcasting and relaying, a new and different LDGM code results, forming a degree-D LDGM ensemble. The on-the-fly construction of each LDGM code in the ensemble is similar to the conventional random construction of a fixed LDGM code, but due to the lack of central control (which is expensive), subsequent adjustment to eliminate short cycles in the code graph is not possible. To show that these rudimentary random LDGM ensembles can nevertheless be efficient, we present in the below several simulation results.

B. Experimental Results

Let us consider a large wireless relay network with $N = 1000$ transmitting terminals and a cooperation level of $\eta = 50\%$ for each terminal. A cooperation level is defined as the ratio between the duration of the relay phase and the broadcast phase. Assuming all the symbols/packets have the same size and are transmitted at the same rate, $\eta = 50\%$ means that each terminal sends one symbol for itself and relays one for others. The resulting ensemble of LDGM codes thus have rate 1/2 and codeword length 2000. To demonstrate the efficiency of the proposed *adaptive network coded cooperation*, we assume relatively noisy inter-user channels, such that each terminal gets only $\rho = 10\%$ the chance to successfully detect the source symbol transmitted by any other terminal. In other words, each terminal will form its relay symbol using D data symbols arbitrarily chosen from a received set of about 100 random candidates.

For simplicity, we assume a homogeneous network where all the user-destination channels follow a frequency-flat quasi-static Rayleigh fading model with the same average quality:

$$y = \alpha x + n, \tag{1}$$

where α is the complex Rayleigh fading coefficient, n is the complex additive white Gaussian noise (AWGN) and $x \in \{-1, +1\}$ is transmitted signal modulated using the binary phase shift keying (BPSK). We further assume that all the transmission channels are spatially independent, and that the channel gain is known to the respective receiver but unknown to the transmitter. Hence, channel state information (CSI) can be exploited to assist decoding, but transmission power allocation is not possible.

For comparison purpose, we also consider a *non-cooperative* case and two decode-and-forward *repetition* cooperative cases. The *fixed repetition* case adopts a similar cooperation rule as described in [4], where each terminal relays only for one of its partners according to a pre-defined rule: e.g. terminal 1 repeats terminal N, terminal 2 repeats terminal 1, and so on. Since a terminal may get an extremely noisy copy of what it needs to forward, making the repeat-and-forward rather useless, we allow for a certain degree of freedom in the relay. We then have the *adaptive repetition* case, where each terminal can choose the data symbol it wishes to forward from its received set, provided that this data symbol has not yet been repeated by others (assume that it hears what others forward.) In both repetition schemes, the relay will back off and stay idle if it does not have a clean copy the data symbol that can be forwarded.

Figure 4 shows the error rate performance of the afore-mentioned LDGM network code ensembles. The X axis denotes the average signal-to-noise ratio (SNR) of the channels from the terminals to the destination. The transmission energy is normalized to account for the rate penalty (rate of the network code). The Y axis denotes the bit error rate (BER) of the LDGM ensemble, which, in the context of user cooperation, corresponds to the outage probability of each source symbol/packet.

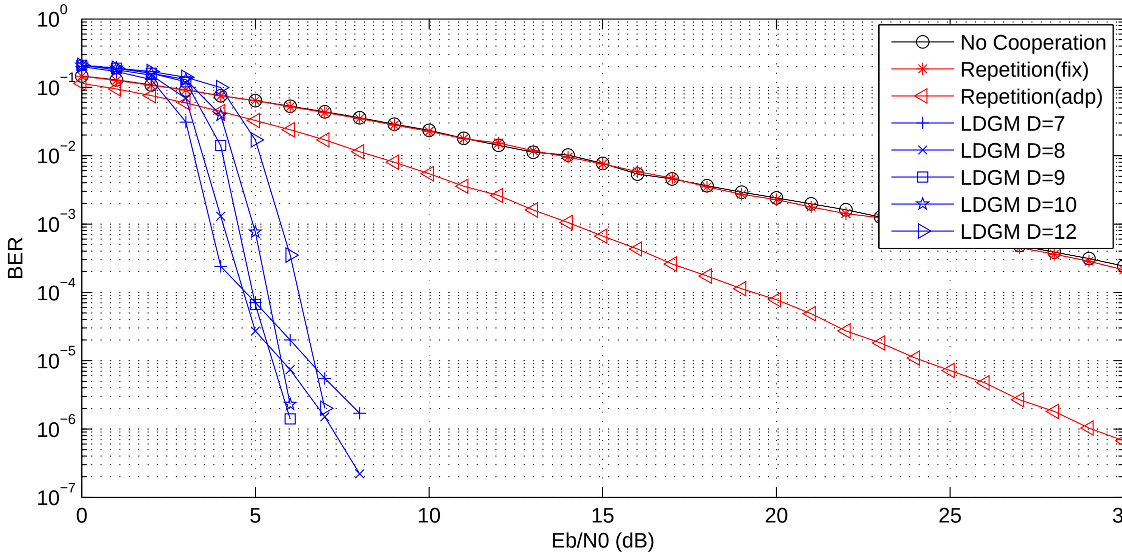

Fig. 4. Performance of *adaptive network coded cooperation* using LDGM ensembles. $N = 1000$ transmitters.

The plot shows that the *fixed repetition* scheme has a marginal improvement over *no cooperation*. In fact, the two curves are hugging each other and hard to differentiate. This should not be surprising, since the fixed forwarding rule and the noisy inter-user channels make each relay have only $\rho = 10\%$ of the chance to help. Hence, although a small portion ($\rho N = 100$) of the data symbols have attained a diversity order of 2, the majority (900) have only a diversity order of 1, making the effective cooperation level (averaged over all terminals) rather low:

$$\text{Fixed Repetition:} \quad \eta^* = \rho N/(N + \rho N) = 0.1/1.1 = 9.09\%. \tag{2}$$

By adjusting the the forwarding rule to match the network topology, *adaptive repetition* can considerably increase the chances of successful relaying. For the tth relay terminal, it finds a good data symbol to repeat with a chance of 100% if $t \leq \rho N$ and $1 - \binom{N-\rho N}{t-1-\rho N}/\binom{N}{t-1}$ if $\rho N < t \leq N$. For example, 100th, 500th, 900th, 950th, 975th, 995th, and 1000th relay terminal has a probability of 100.0%, 100.0%, 100.0%, 99.6%, 93.8%, 47.0% and 10.0%, respectively, to make a successful relay. That is, only in the final few relay attempts, does the probability of successful relaying start to drop. This leads to a significant increase in the effective cooperation level:

$$\text{Adaptive Repetition:} \quad \eta^* = \frac{\rho N + \sum_{t=\rho N+1}^{N} \left[1 - \binom{N-\rho N}{t-1-\rho N}/\binom{N}{t-1}\right]}{N + \rho N + \sum_{t=\rho N+1}^{N} \left[1 - \binom{N-\rho N}{t-1-\rho N}/\binom{N}{t-1}\right]} = 49.78\%, \tag{3}$$

which warrants a successful repetition, and hence a diversity order or 2, for almost all the data packets.

From the network coding perspective, the *adaptive repetition* scheme can be taken as a trivial form of *adaptive network coded cooperation*, where a degenerated LDGM ensemble with degree $D = 1$ is used, i.e. both the left part and the right part of the parity check matrix H are (permutations of) an identity matrix. We expect sophisticated LDGM code ensembles to offer a considerable higher gain. First, the use of non-degenerated LDGM ensembles guarantees an effective cooperation level of

$$\text{Adaptive Network Coded Cooperation:} \quad \eta^* = 50\%, \tag{4}$$

provided that $\rho N >> D$ which is almost always the case since D is a rather small number (typically below 10). More importantly, the large coding gain provided by LDGM code ensembles will translate to a high diversity order, bringing a significant reduction in outage rate compared with either the *non-cooperation* or the *repetition* cases. As shown in Figure 4, the LDGM ensembles can easily provide more than 25 dB gains over the *non-cooperation* and the *fixed repetition* scheme, and some 15 dB gain over the *adaptive repetition* scheme, at an error rate of 10^{-4}!

Furthermore, it is interesting to observe that the performance of a (non-degenerated) LDGM ensemble is closely related to its degree. Each error rate curve of LDGM ensembles can be divided into two regions, the "waterfall" region where the error rate drops quickly, and the "error floor" region where the error rate decreases with a much smaller slope. We see that the larger the degree of the LDGM ensemble, the worse the waterfall performance but the lower the error floor. For example, for an ensemble degree of $D = 7$, it requires only 3.7 dB to achieve an error rate of $P_e = 10^{-3}$, but the error floor starts to appear at a high error level of around 2×10^{-4}. For $D = 8$, it requires some 4.0 dB to achieve $P_e = 10^{-3}$, but the error floor does not appear until a much lower error level of 3×10^{-5}. When the ensemble degree increases to $D = 9$, another 0.4 dB is required to achieve $P_e = 10^{-3}$, but there exhibits no error floor in the region of interest (i.e. error rate above 10^{-6}). The implication of this ensemble performance is that one needs to carefully select the degree for a balanced trade-off between the error floor performance and the waterfall performance. For what we have simulated, we find the degree-9 LDGM ensemble a favorable candidate, which delivers an error rate of 10^{-6} at around 6 dB (see Figure 4).

IV. Lower-Triangular Low-Density Parity-Check Network Codes

A. *Adaptive LT-LDPC Network Codes*

The preceding section presents LDGM code ensembles as a natural candidate for *adaptive network coded cooperation*, applicable for a general (large) network that assigns orthogonal channels to terminals, be it orthogonal in frequency, time or code (such as in a FDMA, TDMA or CDMA system). If the orthogonal channels are time division based, then the causality condition enables the exploitation of an even better code ensemble, namely, lower-triangular LDPC codes.

From the coding theory, one realizes that the identity submatrix in the parity check matrix of an LDGM code makes the message-passing algorithm inefficient. The bits that correspond to these weight-1 columns are the potential cause for error propagations, since the out-bound log-likelihood ratios (LLR) from these bits remain to be the same LLRs computed from the channel observations, and will never get updated or corrected (if they are wrong) during the iterative decoding process. To rectify this, we resort to lower-triangular LDPC codes, another class of linear-time encodable LDPC codes.

As shown in Figure 5, the structure of an LT-LDPC code is similar to that of an LDGM code, except that the right part of the parity check matrix H is a lower triangular sparse matrix with all ones in the main diagonal. Although its generator matrix may be dense, an LT-LDPC code can nevertheless be encoded in linear time using backward substitution.

Lower triangular LDPC code ensembles can be exploited in wireless relay networks in a way similar to LDGM ensembles. The major difference is that in the LDGM case, each terminal collects its set of received symbols only in the broadcast phase, while in the LT-LDPC case, a terminal who is yet to relay will continue to enrich its received set during the relay phase. In other words, when a terminal is forwarding a check-sum symbol to the destination, the remaining terminals listen to what it sends, and add the check-sum symbol to their received sets upon correct reception. In forming a check-sum, a terminal will randomly select symbols from its received set regardless of whether they are source symbols received in the broadcast phase (systematic part) or relay symbols received in the relay phase (parity part).

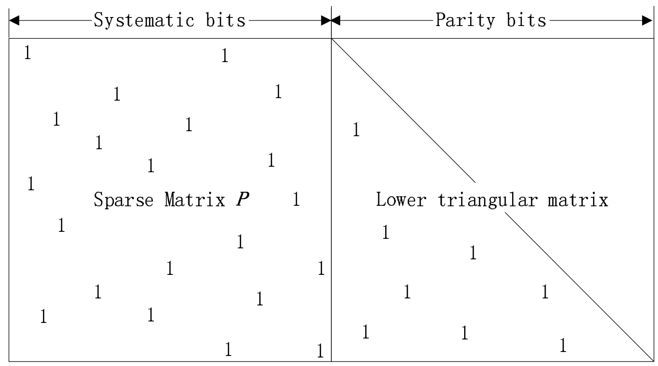

Fig. 5. The parity check matrix of an LT-LDPC code.

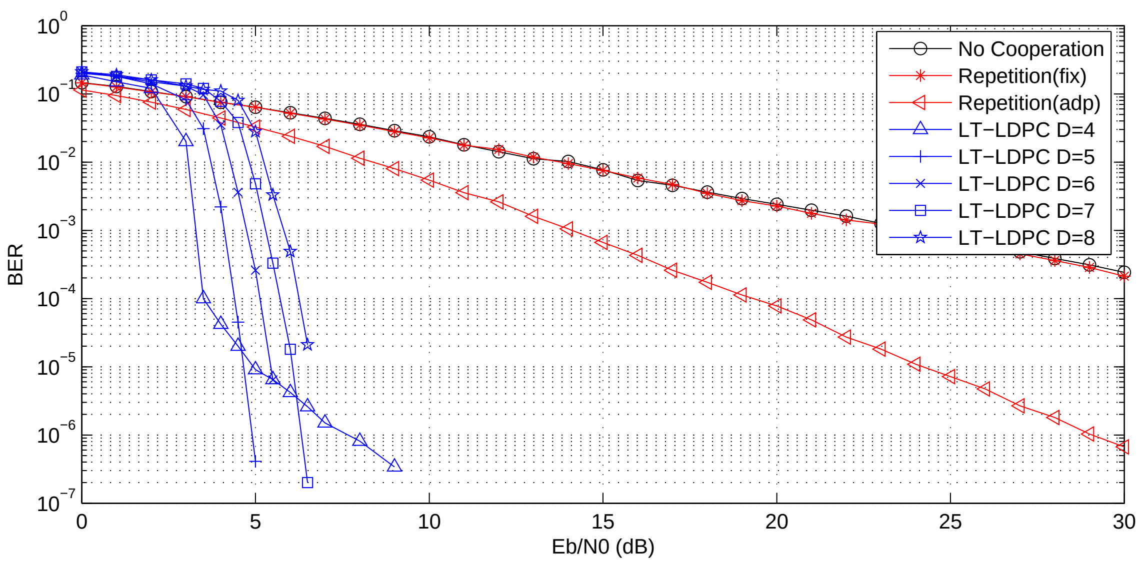

Fig. 6. Performance of *adaptive network coded cooperation* using LT-LDPC ensembles. $N = 1000$ transmitters.

B. Experimental Results

We examine the efficiency of LT-LDPC network codes in the same wireless relay network setup described in Subsection III-B. Again we focus on rate 1/2 code ensembles. For a uniform weight distribution in the non-zero part of the parity check matrix, we adjust the number of symbols participating in each check-sum as follows: the first relay terminal selects D symbols from its received set, and the tth relay terminal selects $(D + \lfloor Dt/N \rfloor)$ symbols, $2 \le t \le N$. We call the resulting LT-LDPC ensemble "degree-D" LT-LDPC ensemble.

Figure 6 shows the simulation results for the *adaptive network coded cooperation* using LT-LDPC code ensembles. As we expect, considerable gains are achieved over the *no cooperation* and the *repetition* schemes. Like LDGM ensembles, the smaller the degree of the LT-LDPC ensemble, the better the waterfall performance; but unlike LDGM ensembles, LT-LDPC ensembles perform much better in error flaring. Only for the ensemble degree of

$D = 4$, do we observe an error floor for the LT-LDPC ensemble; for degree 5 and above, no error floor is found in the region we simulated. Besides a lower error floor, LT-LDPC ensembles also exhibit a (slightly) better waterfall performance than LDGM ensembles. For example, the LT-LDPC ensemble with degree $D = 5$ achieves an error rate of 10^{-6} at less than 5 dB, which is 1 dB better than the best LDGM ensemble.

V. CONCLUSION

The problem of wireless user cooperation in a three-terminal basic scenario has been extensively studied, but cheap and practical solutions for large relay networks are still lacking. In this paper, network coding, a technique originally developed for routing in lossless networks, is shown to be an efficient and practical way to improve the performance of large wireless relay networks. A new framework, termed *adaptive network coded cooperation*, is proposed, where the central idea is to match the instantaneous network graph with the channel code graph on-the-fly. The adaptivity feature of the framework cleverly captures the lossy nature of the wireless media and considerably increases the effective cooperative level. The use of sophisticated network codes over the trivial repetition code further enhances the performance by offering a remarkable coding/diversity gain.

Under this framework, we first suggest to use LDGM ensembles as candidate network codes. The random code structure, the linear-time encodability and the availability of an efficient decoder make LDGM codes a natural fit for *adaptive network coded cooperation*. When the network is TDMA based, lower-triangular LDPC ensembles are an even better choice, with a much lower error floor and a slightly better waterfall performance. Both coding strategies offer a high bandwidth efficiency without the need for strict inter-user synchronization. Simulation results confirm that the new ANCC framework can drastically improves the system performance compared with the existing *repetition* cooperative schemes. When 1000 terminals cooperate together, the adaptive LDGM network codes outperform the *fixed repetition* scheme by more than 40 dB gain and the *adaptive repetition* scheme by as much as 23 dB at an error rate of 10^{-6} ! The adaptive lower-triangular LDPC codes offers a further improvement of 1 dB with no error floors in general.

REFERENCES

[1] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Prob.*, vol. 3, pp. 120-154, 1971.

[2] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 572-584, Sept. 1979

[3] J. N. Laneman, and G. W. Wornell, "Energy-effi cient antenna sharing and relaying for wireless networks," in *Proc. IEEE WCNC,*, 2000, pp. 7-12.

[4] J. N. Laneman, and G. W. Wornell, "Distributed Space-Time-Coded protocols for exploiting cooperative diversity in wireless networks," in *IEEE Trans. Inform. Theory*, vol. 49, NO. 10, Oct 2003, pp. 2415-2425.

[5] X. Bao, M. Yu, and J. Li, "A new user cooperation scheme for uplink wireless networks," *Proc. Allerton Conf. on Commun., Control and Computing* Urbana Champaign, IL, Sept. 2004.

[6] T. E. Hunter and A. Nosratinia, "Cooperation diversity through coding," in *Proc. Int. Symp. Inform. Theory*, Laussane, Switzerland, 2002, pp. 220-221.

[7] T. E. Hunter and A. Nosratinia, "Space-time diversity through coded cooperation," in *IEEE Journal on Select Areas in Communications*, 2003, submitted for publication.

[8] M. Janani, A. Hedayat, T. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," in *IEEE Transactions on Signal Processing*, vol. 52, NO. 2, Feb 2004, pp. 362-371.

[9] R. Ahlswede, N. Cai, S.-Y.R. Li and R.W. Yeung, "Network information fbw", *IEEE Trans. Inform. Theory*, vol. 46, pp. 1204-1216, 2000.

[10] http://tesla.csl.uiuc.edu/∼koetter/NWC/

[11] J. Garcia-Frias, and Wei Zhong, "Approaching Shannon performance by iterative decoding of linear codes with low-density generator matrix," *IEEE Commun. Letters*, pp. 266-268, June 2003.

[12] M. Luby, "LT codes," *Proc. IEEE Symp. on the Foundations of Computer Science*, pp. 271-280, 2002.

[13] A. Shokrollahi, "Raptor codes," *http://www.inference,phy.cam.ac.uk/mackay/DFountain.html*