Error Rate Performance of Coded Free-Space Optical Links over Gamma-Gamma Turbulence Channels

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***Abstract*—Error control coding can be used over free-space optical (FSO) links to mitigate turbulence-induced fading. In this paper, we derive error performance bounds for coded FSO communication systems operating over atmospheric turbulence channels, considering the recently introduced gamma-gamma turbulence model. We derive a pairwise error probability (PEP) expression and then apply the transfer function technique in conjunction with the derived PEP to obtain upper bounds on the bit error rate. Simulation results are further demonstrated to confirm the analytical results.**

***Keywords*—Atmospheric turbulence channel, free-space optical communication, pairwise error probability, error performance analysis.**

I. INTRODUCTION

Wireless optical communications, also known as free-space optical (FSO) communications, is a cost-effective and high bandwidth access technique and is receiving growing attention with recent commercialization successes [1]. With the potential high-data-rate capacity, low cost and particularly wide bandwidth on unregulated spectrum, FSO communications is an attractive solution for the "last mile" problem to bridge the gap between the end user and the fiber-optic infrastructure already in place. Its unique properties make it also appealing for a number of other applications, including metropolitan area network extensions, enterprise/ local area network connectivity, fiber backup, back-haul for wireless cellular networks, redundant link and disaster recovery. In FSO communications, optical transceivers communicate directly through the air to form point-to-point line-of-sight links. One major impairment over FSO links is the atmospheric turbulence, which occurs as a result of the variations in the refractive index due to inhomogeneties in temperature and pressure fluctuations. The atmospheric turbulence results in fluctuations at the received signal, i.e. signal fading, also known as scintillation in optical communication terminology [2], severely degrading the link performance, particularly over link distances of 1 km or longer.

Error control coding as well as diversity techniques can be used over FSO links to improve the error rate performance [3-6]. In [6], Zhu and Kahn studied the performance of coded FSO links assuming a log-normal channel model for atmospheric turbulence. Specifically, they derived an approximate upper bound on the pairwise error probability (PEP) for coded FSO links with intensity modulation/direct direction (IM/DD) and provided upper bounds on the bit error rate (BER) using the transfer function technique. Although log-normal distribution is the most widely used model for the probability density function (pdf) of the irradiance due to its simplicity, this pdf model is only applicable to weak turbulence conditions [2]. As the strength of turbulence increases, multiple scattering effects must be taken into account. In such cases, lognormal statistics exhibit large deviations compared to experimental data. Furthermore, it has been observed that lognormal pdf underestimates the behavior in the tails as compared with measurement results. Since detection and fade probabilities are primarily based on the tails of the pdf, underestimating this region significantly affects the accuracy of performance analysis. Due to the limitations of log-normal model, many statistical models have been proposed over the years to describe atmospheric turbulence channels under a wide range of turbulence conditions, e.g. *K* distribution, *I-K* distribution, and log-normal Rician channel [2]. Error rate performance of coded FSO links assuming *K* distribution and *I-K* distribution have been already studied by the authors in [7].

In a recent series of paper on scintillation theory [8-10], Andrews et.al. introduced the *modified Rytov theory* and proposed *gamma-gamma* pdf as a tractable mathematical model for atmospheric turbulence. This model is a two-parameter distribution which is based on a doubly stochastic theory of scintillation and assumes that small-scale irradiance fluctuations are modulated by large-scale irradiance fluctuations of the propagating wave, both governed by independent gamma distributions. The gamma-gamma pdf can be directly related to atmospheric conditions and provides a good fit to experimental results.

In this paper, we will investigate error rate performance of coded FSO links operating over atmospheric channels, where the turbulence-induced fading is described by the gamma-gamma distribution. The organization of the paper is as follows: In Section II, we review the gamma-gamma channel model under consideration. In Section III, an approximate PEP expression is derived for an FSO communication system with on-off keying (OOK). In Section IV, we present numerical results to demonstrate the accuracy of the derived PEP in comparison to the exact PEP. Using transfer function technique in conjunction with the derived PEP expression, we also obtain bounds on the BER performance. Analytical results are further confirmed through Monte-Carlo simulation. Conclusions are presented in Section V.

II. ATMOSPHERIC TURBULENCE CHANNEL MODEL

In the classical Rytov theory [2], the optical field is represented by

$$U(r,L)=U_0(r,L)\exp\left[\sum_i \psi_i(r,L)\right] \quad (1)$$

where r is the observation point in transverse plane at propagation distance L, $U_0(r,L)$ is the optical field in the absence of turbulence and $\sum_i \psi_i(r,L)$ is the total complex phase perturbation of the field due to random inhomonegeties along the propagation path. Here, $\psi_i(r,L)=\chi_i+j\xi_i$ $i=1,2,..$ represent i^{th}-order perturbations, where $\chi_i(r,L)$ and $\xi_i(r,L)$ are the corresponding real and imaginary parts, respectively. Early studies in modeling the atmospheric turbulence consider a first-order Rytov approximation, i.e.

$$U(r,L)=U_0(r,L)\exp[\chi_1(r,L)+j\xi_1(r,L)] \quad (2)$$

and assume the perturbation $\psi_1(r,L)=\chi_1(r,L)+j\xi_1(r,L)$ is a complex Gaussian random process. The irradiance of the random field takes the form

$$I=|U_0|^2\exp(2\chi_1) \quad (3)$$

where $|U_0|$ is the amplitude of the unperturbed field. Using the first-order Rytov approximation, the irradiance is modeled as a log-normal distribution. This is fairly accurate for describing weak turbulence, but incurs a large discrepancy in strong turbulence case due to the underweighing of multiple scattering effect. A common practice to mitigate this discrepancy is to employ a second-order Rytov approximation where the optical field and the corresponding irradiance are given by,

$$U(r,L)=U_0(r,L)\exp[\chi_1(r,L)+\chi_2(r,L)+j(\xi_1(r,L)+\xi_2(r,L))]$$

and $I=\left(|U_0|^2\exp(2\chi_1)\right)\exp(2\chi_2)$ (5)

respectively. Here the second term $\exp(2\chi_2)$ acts like a random modulation of the first term. The implication of (5) is that the irradiance can be described by a modulation process involving first-order and second-order log-amplitude modulation perturbations. However, this does not lead to a simple model where the perturbation terms can be related to atmospheric parameters. Alternative models where the irradiance fluctuation is modeled as the result of two multiplicative random processes include the *Rician/Log-normal* model [11], the *Nakagami/Gamma* model [12] and the *Negative Exponential/Gamma* model (also known as the K channel) [13] among others.

More recently, Andrews et.al. proposed the *modified Rytov theory* [8-10], which defines the optical field as

$$U(r,L)=U_0(r,L)\exp[\Psi_x(r,L)+\Psi_y(r,L)] \quad (6)$$

where $\Psi_x(r,L)$ and $\Psi_y(r,L)$ are statistically independent complex perturbations which are due only to large-scale and small-scale atmospheric effects, respectively. Put another way, the irradiance is now defined as the product of two random processes, i.e. $I=I_xI_y$, where I_x arises from large scale turbulent eddies and I_y from small-scale eddies. Specifically, in [10], gamma pdf is used to model both small-scale and large-scale fluctuations, leading to the so-called *gamma-gamma* pdf, i.e.

$$f(I)=\frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}I^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right),\ I>0 \quad (7)$$

where $K_a(.)$ is the modified Bessel function of the second kind of order a. Here, α and β are the effective number of small-scale and large scale eddies of the scattering environment. These parameters can be directly related to atmospheric conditions according to [10]

$$\alpha=\left[\exp\left(\frac{0.49\beta_0^2}{\left(1+0.18d^2+0.56\beta_0^{12/5}\right)^{7/6}}\right)-1\right]^{-1} \quad (8)$$

$$\beta=\left[\exp\left(\frac{0.51\beta_0^2}{\left(1+0.9d^2+0.62d^2\beta_0^{12/5}\right)^{5/6}}\right)-1\right]^{-1} \quad (9)$$

where $\beta_0^2=0.5C_n^2k^{7/6}L^{11/6}$ and $d=\left(kD^2/4L\right)^{1/2}$. Here, $k=2\pi/\lambda$ is the optical wave number, λ is the wavelength and D is the diameter of the receiver collecting lens aperture.

C_n^2 stands for the index of refraction structure parameter and is altitude-dependent. Several C_n^2 profile models are available in the literature, but the most commonly used is the Hufnagle-Valley model described by [2]

$$\begin{aligned} C_n^2(h) &= 0.00594(v/27)^2\left(10^{-5}h\right)^{10}\exp(h/1000) \\ &+ 2.7\times10^{-6}\exp(-h/1500) + A\exp(-h/1000) \end{aligned} \quad (10)$$

where h is the altitude in meters (m), v is the rms windspeed in meters per second (m/sec) and A is a nominal value of $C_n^2(0)$ at the ground in m$^{-2/3}$. For FSO links near the ground, C_n^2 can be taken approximately 1.7×10^{-4} m$^{-2/3}$ during daytime and 8.4×10^{-15} m$^{-2/3}$ at night. In general, C_n^2 varies from 10^{-13} m$^{-2/3}$ for strong turbulence to 10^{-17} m$^{-2/3}$ for weak turbulence with 10^{-15} m$^{-2/3}$ often defined as a typical average value [14].

III. Derivation Of PEP

We consider intensity modulation and direct detection links using on-off keying (OOK). Following [6], we assume that the receiver signal-to-noise ratio is limited by shot noise caused by ambient light which is much stronger than the desired signal and/or by thermal noise. In this case, the noise can be modeled as additive white Gaussian noise (AWGN) with zero mean and variance $N_0/2$, independent of the on/off state of the received bit.

The PEP represents the probability of choosing the coded sequence $\hat{\boldsymbol{X}} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_M)$ when indeed $\boldsymbol{X} = (x_1, x_2, ..., x_M)$ was transmitted. Here, we assume that the turbulence-induced fading remains constant over one bit interval and changes from one interval to another in an independent manner. Such an assumption can be justified by the use of perfect interleaving. Under the assumption of maximum likelihood soft decoding with perfect channel state information (CSI), the conditional PEP with respect to fading coefficients $\boldsymbol{I} = (I_1, I_2, ..., I_M)$ is given as [6]

$$P\left(\boldsymbol{X}, \hat{\boldsymbol{X}} \middle| \boldsymbol{I}\right) = Q\left(\sqrt{\frac{\varepsilon\left(\boldsymbol{X}, \hat{\boldsymbol{X}}\right)}{2N_0}}\right) \quad (11)$$

where $Q(.)$ is the Gaussian-Q function and $\varepsilon\left(\boldsymbol{X}, \hat{\boldsymbol{X}}\right)$ is the energy difference between two codewords. Since OOK is used, the receiver would only receive signal light subjected to fading during on-state transmission. Thus, we have

$$P\left(\boldsymbol{X}, \hat{\boldsymbol{X}} \middle| \boldsymbol{I}\right) = Q\left(\sqrt{\frac{E_s}{2N_0}\sum_{k\in\Omega} I_k^2}\right) \quad (12)$$

where E_s is the total transmitted energy and Ω is the set of bit intervals' locations where $\boldsymbol{X}$ and $\hat{\boldsymbol{X}}$ differ from each other. Defining the signal-to-noise ratio as $\tau = E_s/N_0$ and using the alternative form for Gaussian-Q function [15], i.e. $Q(x) = (1/2\pi)\int_0^{\pi/2}\exp\left(-x^2/\sin^2\theta\right)$, we obtain

$$P\left(\boldsymbol{X}, \hat{\boldsymbol{X}} \middle| \boldsymbol{I}\right) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{k\in\Omega}\exp\left(-\frac{\tau}{4}\frac{I_k^2}{\sin^2\theta}\right)d\theta. \quad (13)$$

To obtain unconditional PEP, we need to take an expectation of (13) with respect to I_k. Using independency among fading coefficients I_k, we write

$$\begin{aligned} P\left(\boldsymbol{X}, \hat{\boldsymbol{X}}\right) &= \frac{1}{\pi}\int_0^{\pi/2}\prod_{k\in\Omega}\mathrm{E}_{I_k}\left[\exp\left(-\frac{\tau}{4}\frac{I_k^2}{\sin^2\theta}\right)\right]d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left[\int_0^{\infty}\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta}\right)f(I)dI\right]^{|\Omega|}d\theta \end{aligned} \quad (14)$$

where E(.) is the expectation operation and $|\Omega|$ is the cardinality of Ω, which also corresponds to the length of error event. Here, $f(I)$ is the pdf for the gamma-gamma channel given by (7). A direct use of (7) in (14) yields an expression which unfortunately does not have a closed form solution. To get around with this, we exploit the fact that the underlying distribution is a *conditional* gamma distribution with its mean μ following again a gamma distribution, and rewrite (14) as

$$P\left(\boldsymbol{X}, \hat{\boldsymbol{X}}\right) = \frac{1}{\pi}\int_0^{\pi/2}\left\{E_\mu\left\{E_{I|\mu}\left[\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta}\right)\right]\right\}\right\}^{|\Omega|}d\theta \quad (15)$$

For the gamma-gamma channel, the inner expectation in (15) gives

$$E_{I|\mu}\left[\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta}\right)\right] = \frac{\beta^\beta}{\mu^\beta\Gamma(\beta)}\int_0^\infty I^{\beta-1}\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta} - \frac{\beta I}{\mu}\right)dI$$

Using the result from [16, p.1093, Eq. 2.33], i.e.

$$\int_0^\infty z^{\nu-1}\exp\left(-az^2 - bz\right)dz = (2a)^{-\nu/2}\Gamma(\nu)\exp\left(\frac{b^2}{8a}\right)D_{-\nu}\left(\frac{b}{\sqrt{2a}}\right)$$

where $D_p(.)$ is the parabolic cylinder function, we obtain

$$\begin{aligned} &E_{I|\mu}\left[\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta}\right)\right] \\ &= \frac{\beta^\beta}{\mu^\beta}\left(\frac{\tau}{2}\frac{1}{\sin^2\theta}\right)^{-\beta/2}\exp\left(\frac{\beta^2\sin^2\theta}{2\mu^2\tau}\right)D_{-\beta}\left(\frac{\sqrt{2}\beta}{\mu\sqrt{\tau}}\sin\theta\right) \end{aligned} \quad (16)$$

Since the operation of expectation over μ does not yield a closed form, we resort to the asymptotic expansion of the parabolic cylinder function given as [17- p. 689, Eq.19.9]

$$D_{-\left(a+\frac{1}{2}\right)}(z) = \frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{4}+\frac{a}{2}\right)}2^{-\left(\frac{a}{2}+\frac{1}{4}\right)}\exp\left(-\left(\sqrt{a}+\frac{1}{16}a^{-3/2}\right)z - \mathrm{O}\left(z^2\right)\right)$$

where $O(z^n)$ represents the terms with power equal or higher than n. The above holds for $z^2 << a$ and this condition is easily satisfied in our case for high SNR values. Replacing the asympotic expression in (16) and using the resulting expression in (15), we have

$$P(\boldsymbol{X},\hat{\boldsymbol{X}})=\frac{1}{\pi}\int_0^{\pi/2}\left[c_1\tau^{-\frac{\beta}{2}}(\sin\theta)^{\beta}\int_0^{\infty}\mu^{\alpha-\beta-1}\right. \tag{17}$$

$$\left.\times\exp\left(-\alpha\mu-c_2\frac{\sqrt{2}\sin\theta}{\sqrt{\tau}}\mu^{-1}-O(\mu^{-2})\right)d\mu\right]^{|\Omega|}d\theta$$

where

$$c_1=\frac{\sqrt{\pi}\alpha^{\alpha}\beta^{\beta}}{\Gamma(\alpha)\Gamma\left(\frac{\beta+1}{2}\right)}\ ,\ c_2=\beta\left(\sqrt{\beta-\frac{1}{2}}+\frac{1}{16}\left(\beta-\frac{1}{2}\right)^{-\frac{3}{2}}\right) \tag{18}$$

Neglecting the higher order components in (17), the inner integral can be solved with the help of [16- p. 384, Eq. 3.471.9]

$$\int_0^{\infty}z^{v-1}\exp\left(-az-b\frac{1}{z}\right)dz=2\left(\frac{b}{a}\right)^{v/2}K_v\left(2\sqrt{ab}\right),\ a>0,b>0\,.$$

This yields the final form for PEP as

$$P(\boldsymbol{X},\hat{\boldsymbol{X}})\cong\frac{1}{\pi}\int_0^{\pi/2}\left[2^{\frac{\alpha-\beta+4}{4}}c_1\left(\frac{c_2}{\alpha}\right)^{\frac{\alpha-\beta}{2}}\left(\frac{\sin\theta}{\sqrt{\tau}}\right)^{\frac{\alpha+\beta}{2}}\right. \tag{19}$$

$$\left.\times K_{\alpha-\beta}\left(2^{5/4}\sqrt{\frac{c_2\alpha\sin\theta}{\sqrt{\tau}}}\right)\right]^{|\Omega|}d\theta$$

It should be emphasized that (19) is an approximation since the higher order components in the asymptotic expansion of the parabolic cylinder function are neglected. A union bound on the average BER can be found as [15]

$$P_b\le\frac{1}{\pi}\int_0^{\pi/2}\left[\frac{1}{n}\frac{\partial}{\partial N}T(D(\theta),N)\bigg|_{N=1}\right]d\theta\,, \tag{20}$$

where N is an indicator variable taking into account the number of bits in error and $D(\theta)$ is defined based on the underlying PEP expression.

IV. NUMERICAL RESULTS

In this section, we will first compare the derived approximate PEP with the exact PEP expression. Then, as an example, we will consider a convolutionally coded FSO system and will use our PEP results to compute upper bounds on the BER of the considered system.

In Fig. 1, we plot the derived PEP approximation given by (19) for an error event of length 3, i.e. $|\Omega|=3$, using different values of channel parameters α and β. We also compute the exact PEP given by (14) using numerical integration and include it in the figures as a reference. It is observed from both figures that the derived PEP provides a good approximation and coincides with the exact PEP for high signal-to-noise ratios. As a result of the neglected higher order terms in the approximation of parabolic cylinder function, the derived PEP behaves as a lower bound for the considered cases.

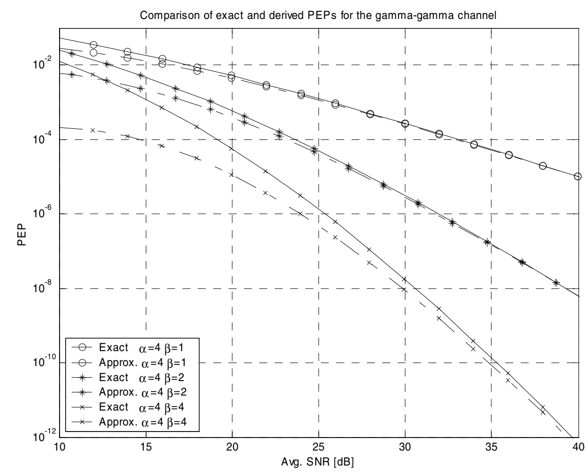

Fig. 1. Comparison of exact and derived PEPs for α=4 and β=1, 2, 4 (solid: exact, dashdot: derived)

In the following, we consider an FSO communication system operating at $\lambda=1550\text{nm}$. We assume $C_n^2=1.7\times10^{-4}$ a typical value of refraction index for FSO links near the ground during daytime. A point receiver is used, i.e. $D<<L$ leading to d=0, therefore no aperture averaging is possible and system performance relies heavily on possible coding gains. We use a convolutional code, which has a code rate of 1/3 and constraint length of 3 [18-Fig.8.2.2]. The transfer function of this code is found to be

$$T(D(\theta),N)=D^6(\theta)N/\left(1-2ND^2(\theta)\right) \tag{21}$$

Replacing (21) in (20), we obtain

$$P_b\le\frac{1}{\pi}\int_0^{\pi/2}\frac{D^6(\theta)}{\left(1-2D^2(\theta)\right)^2}d\theta \tag{22}$$

where $D(\theta)$ is given by

$$D(\theta)=2^{\frac{\alpha-\beta+4}{4}}c_1\left(\frac{c_2}{\alpha}\right)^{\frac{\alpha-\beta}{2}}\left(\frac{\sin\theta}{\sqrt{\tau}}\right)^{\frac{\alpha+\beta}{2}}K_{\alpha-\beta}\left(2^{5/4}\sqrt{\frac{c_2\alpha\sin\theta}{\sqrt{\tau}}}\right) \tag{23}$$

based on the derived PEP with c_1 and c_2 defined as in (18). In the case of exact PEP, $D(\theta)$ is given by

$$D(\theta)=\int_0^{\infty}\frac{2(\alpha\beta)}{\Gamma(\alpha)\Gamma(\beta)}I^{\frac{\alpha+\beta}{2}-1}\exp\left(-\frac{\tau}{4}\frac{I^2}{\sin^2\theta}\right)K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right)dI \tag{24}$$

The average BER results are computed based on (22) in conjunction with (23) as well as with (24) to allow comparison with the true upper bound. Both of them are illustrated in the Fig. 2 for the FSO scenario under investigation. Specifically, we present BERs for three different link distances L=2600m, L=3000m and L=3400m, which yield the channel parameters $(\alpha,\beta)=(3.79,2.41)$, $(\alpha,\beta)=(2.73,1.25)$, $(\alpha,\beta)=(2.26,0.65)$, respectively. For all three cases we considered, BER estimates based on the derived PEP yield a very good approximation to the true upper bound. Although there is some discrepancy in the lower SNR region, it provides excellent agreement as SNR increases. Monte-Carlo simulation results are furthermore included as a reference. Due to the long simulation time involved, we are able to give simulation results only up to BER=10^{-6}. Simulation results are observed to be located slightly lower than the true upper bound and demonstrate an excellent agreement with the analytical results. Considering BER=10^{-9} is a practical performance target for an FSO link, our analytical results can serve as a simple and reliable method to estimate BER performance without resorting to lengthy simulations.

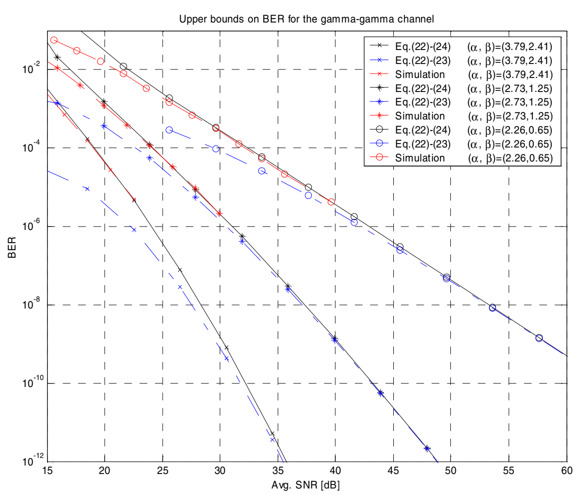

Fig. 2. Upper bounds on BER for the gamma-gamma channel (solid black: Eq. (22)-(24), dashed dot blue: Eq. (22)-(23), dashed red: Simulation)

V. CONCLUSIONS

In this paper, we investigate error rate performance of coded FSO systems operating over atmospheric turbulence channels, which are modeled with gamma-gamma distribution. Unlike the classically used log-normal assumption which is only accurate for modeling weak turbulence, the gamma-gamma channel model works for a variety of turbulence conditions. A PEP approximation is derived for the gamma-gamma channel and the transfer function technique is employed to obtain upper bounds on the BER performance of coded FSO links with OOK. Simulation results are also included to confirm the analytical results.

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