On the Performance of Turbo Product Codes over Partial Response Channels

Jing Li, Erozan Kurtas, Krishna R. Narayanan, and Costas N. Georghiades

***Abstract*—This paper evaluates the performance of single-parity check turbo product codes (TPC/SPC) over partial response channels. A rate-0.94 and a rate-0.89 TPC/SPC code are considered for use with PR4/EPR4 channels with proper precoding and with turbo equalization. Gains of 4.5 to 5 dB are obtained at BER of 10^{-5}, revealing performance comparable to that of low density parity check codes. Apart from its linear encoding/decoding complexity and highly parallelizable decoding algorithm, TPC/SPC codes demonstrate favorable error statistics which are in harmony with the outer Reed–Solomon error correction code (RS-ECC), indicating it to be a promising candidate for future recording systems.**

***Index Terms*—Data storage system, iterative decoding, message-passing decoding, partial response channels, precoding, turbo product codes.**

I. INTRODUCTION

ITERATIVE decoding is being seriously considered for application in future magnetic recording systems. After being precoded, filtered and equalized to some simple partial response (PR) target, the magnetic recording channel can be modeled as an inter-symbol interference (ISI) channel and, hence, turbo equalization has been shown to provide good performance [2].

Turbo codes and low density parity check (LDPC) codes have demonstrated impressive coding gains on magnetic recording channels [1]–[3]. But a turbo decoder uses a complex MAP (maximum *a posteriori*) decoder with many states. An LDPC code has considerably less decoding complexity, but its encoding is $(O(n^2))$. Further, it has been seen that large error bursts within a block may easily cause failure of the outer Reed–Solomon error correction code (RS-ECC) [5].

Single-parity check turbo product codes (TPC/SPC) which are a simple type of turbo product codes (TPC) [4] possess many inviting properties for recording systems, such as high-rate, linear encoding/decoding complexity and a highly parallelizable encoding/decoding procedure. While turbo and LDPC codes have been under extensive investigation in data storage research, little has been reported about TPC/SPC codes in this area. This paper will step through the critical issues of TPC/SPC codes as applied to PR channels and highlight new results. The fact that single-parity check (SPC) codes are intrinsically weak codes and that TPC/SPC codes are worse than LDPC codes over AWGN channels tend to indicate their inferiority over PR channels also. However, when used as an outer code in a serial concatenation with an interleaver and a precoded channel (collectively viewed as a recursive inner code), a significant interleaving gain results. In particular, gains of 4.5–5 dB are observed for a rate-0.94 and a rate-0.89 TPC/SPC code over ideal PR4/EPR4 channels, comparable to that of LDPC codes, yet with less complexity. Further, error statistics show that TPC/SPC codes tend to work in harmony with the outer RS-ECC codes, which possibly makes them a more promising candidate than LDPC codes for future magnetic recording applications.

The paper is organized as follows. Section II addresses the fundamentals of TPC/SPC codes. Section III presents the system model with discussion on precoding and the iterative approach. Section IV reports simulation results. Finally Section V concludes the paper.

II. FUNDAMENTALS OF TPC/SPC CODES

A. Introduction

A turbo product code [4], also known as a block turbo code (BTC), is composed of multi-dimensional arrays of codewords from linear block codes, upon which an iterative soft-in soft-out (SISO) decoding is employed. A 2-dimensional turbo product code, $\mathcal{C}$, formed from component codes $\mathcal{C}_i \sim (n_i, k_i, d_i, G_i)$, $i = 1, 2$, has parameters $(n_1 n_2, k_1 k_2, d_1 d_2, G_1 \otimes G_2)$, where n, k, d, and G denote the codeword size, user data size, minimum distance and generator matrix, respectively, and $\otimes$ denotes the Kronecker product. A popular way to interpret a TPC code is to treat it as a serial concatenation of its component codes with linear block interleaver in between. However a TPC/SPC code can also be viewed as a special type of structured LDPC codes, where each row in each dimension satisfies a check [5]. Hence while the general treatment of a TPC code is via the Chase algorithm, a TPC/SPC code can be decoded using a simple graph-based message-passing algorithm similar to that of an LDPC code. To the interest of recording systems where high-rate codes are required, we focus on 2-d TPC/SPC codes only, but the above properties as well as the decoding algorithm are easily extended to the multi-dimensional case.

B. Decoding Algorithm

Assuming even-parity check codes, bipolar modulation and AWGN channels, a 2-D TPC/SPC code formed from $(N_1, N_1 - 1) \otimes (N_2, N_2 - 1)$ has the following SISO decoding algorithm (Table I), where $Lo_{i,j}$ denotes the *a priori* information, $LLR_{i,j}$ the log-likelihood ratio, and $r_{i,j}$ the observed signal from the channel.

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TABLE I
DECODING ALGORITHM FOR 2-D TPC/SPC CODES

Initialization:
for $i = 1$ to N_2, for $j = 1$ to N_1,
$Lo_{i,j} = \frac{2}{\sigma^2} r_{i,j}, \quad Le_{i,j}^{(1)} = Le_{i,j}^{(2)} = 0,$

Iterations:
Decoding $\mathcal{C}_1$: for $i = 1$ to N_2, for $j = 1$ to N_1,
$Le_{i,j}^{(1)} = 2\tanh^{-1}(\prod_{1 \le t \le N_1, t \ne j} \tanh(\frac{Lo_{i,t} + Le_{i,t}^{(2)}}{2})),$
Decoding $\mathcal{C}_2$: for $j = 1$ to N_1, for $i = 1$ to N_2,
$Le_{i,j}^{(2)} = 2\tanh^{-1}(\prod_{1 \le t \le N_2, t \ne i} \tanh(\frac{Lo_{t,j} + Le_{t,j}^{(1)}}{2})),$

Soft output and decision:
for $i = 1$ to N_2, for $j = 1$ to N_1,
$LLR_{i,j} = Lo_{i,j} + Le_{i,j}^{(1)} + Le_{i,j}^{(2)},$
$\hat{s}_{i,j} = LLR_{i,j} > 0\ ?\ 0 : 1;$

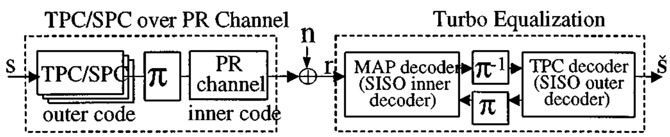

Fig. 1. System model of TPC/SPC codes over PR channels.

III. TPC/SPC CODES OVER PR CHANNELS

A. System Model and Turbo Equalization

We study an ideal system model where the recording channel impulse is modeled as a perfectly equalized partial response polynomial (PR4 or EPR4) with additive white Gaussian noise (Fig. 1). It should be noted that the PR4 channel is not penalized for noise-boosting at high frequencies. This approach has been used in several other papers including [1] and [3]. The observation that PR channels can be effectively viewed as a rate-1 convolutional code leads to a natural form of serial concatenated structure with the ISI channel considered as the inner code and TPC/SPC code as the outer code. With proper precoding, this inner convolutional code appears recursive, providing the potential for interleaving gain. Turbo equalization [also known as iterative decoding and equalization (IDE)], is exploited to iterate soft extrinsic information in log-likelihood ratio (LLR) form between the inner and the outer code. The random interleaver in between works to decorrelate LLRs and to eliminate error bursts. Each turbo iteration (big loop) in TPC/SPC system composes of one round of inner MAP decoding (implemented using the BCJR algorithm), followed by 2 rounds of iterations in outer TPC/SPC decoder (small loop).

B. Precoder and Distance Spectrum

A precoder typically takes the form of $1/g(D)$ where $g(D)$ is a polynomial in the binary field. Precoding makes the inner code (i.e., ISI channel) appear recursive, and will (in conjunction with the random interleaver) improve the distance spectrum of the overall code (spectral thinning) by mapping low weight error events to higher weight ones, leading to an interleaving gain. It has been shown that the minimum distance of the outer code should be at least 3 [6] in order to obtain an interleaving gain.

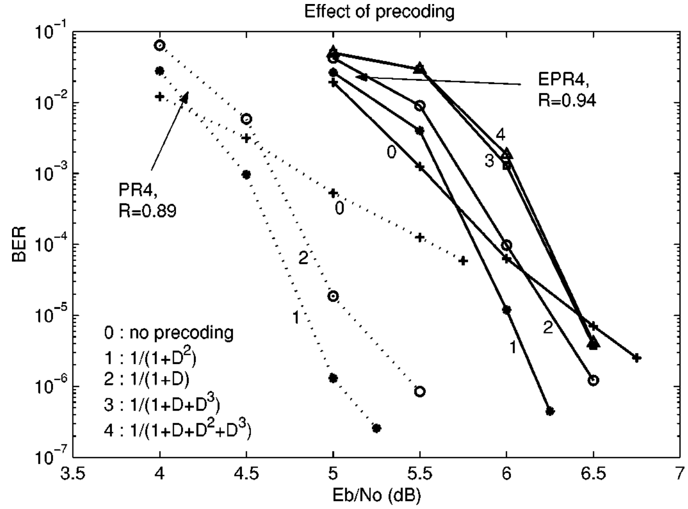

Fig. 2. Effect of precoding.

Since TPC/SPC codes have a minimum distance of $d_m = 4$ even for very high rates, an interleaving gain results. It should be noted that a high-rate punctured convolutional code usually has $d_m \le 3$ and, hence, TPC/SPC codes can be expected to outperform high-rate convolutional codes. In comparison to an LDPC code, since a randomly-constructed LDPC code generally has a very large minimum distance, precoding cannot bring further (effective) spectral thinning. It has been shown in [7] that the performance of LDPC codes (with average column weight 3) is worse with precoding than without precoding, so for a fair comparison, no precoding is used for LDPC codes in this work. A 2-D TPC/SPC code has minimum distance of only 4 and therefore encounters many undetectable errors. Several techniques have been proposed to improve the distance spectrum of TPC/SPC codes. However, they usually either decrease the code rate or increase the complexity and, hence, are not of interest.

In addition to enhancing the distance spectrum, the precoder also affects the convergence of the turbo equalization process and, hence, the precoder should be carefully chosen. In particular, the best precoder for PR4/EPR4 channels is shown to be $1/(1 \oplus D^2)$ [7], and our simulation confirms the claim (Fig. 2).

IV. SIMULATION RESULTS

Simulation Parameters: We study 2-d TPC/SPC codes with rate 0.89 and 0.94 which are formed from (17,16) and (33,32) SPC codes, respectively. We combine 16 $(17, 16)^2$ TPC/SPC codewords and 4 $(33, 32)^2$ TPC/SPC codewords respectively to form an effective data block size of 4K bits, to obtain a larger interleaving gain.

Bit Error Rate: As shown in Fig. 3, gains of some 4.5–5 dB over uncoded partial response maximum likelihood (PRML) systems are observed for TPC/SPC codes at BER of 10^{-5}. Also presented are curves for a rate $8/9 = 0.89$ and $16/17 = 0.94$ regular LDPC codes with column weight 3. It should be noted here that irregular LDPC codes of such high rates have been seen to perform slightly worse than regular codes [8]. Precoding is used for TPC/SPC codes and no precoding for LDPC codes. Although not shown, a rate-0.94 LDPC is seen to be 0.5 dB worse with than without precoding on PR4 channels at BER of 10^{-3}.

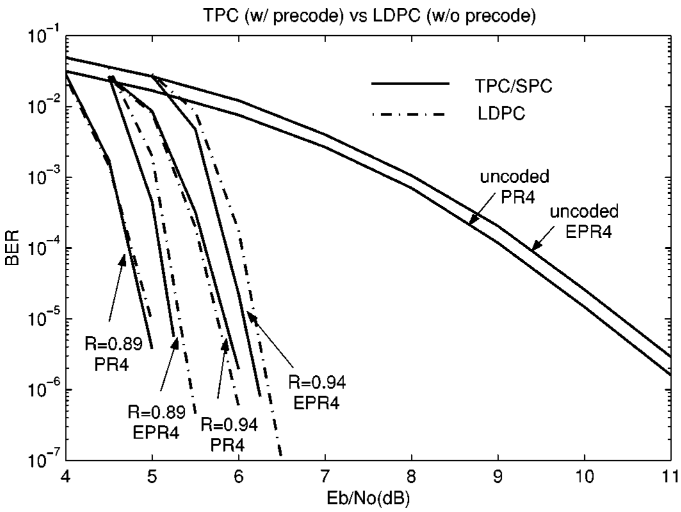

Fig. 3. Performance of TPC/SPC versus LDPC.

Block Error Pattern: Magnetic recording systems require BER of less than 10^{-15}. Such low error rate is achieved by concatenating an RS-ECC at the very end to clear up the residual errors. A typical RS-ECC works on the byte level, capable of correcting up to t byte errors in each data block of size 4K bits or 512 bytes (t is usually around 10 to 20). Hence the maximum number of uncorrected errors after the modulation code in each block should be small. Whereas LDPC codes are shown to have undesirable bursty errors [5], TPC/SPC codes appear to have nicer error statistics. Fig. 4 compares the error statistics for a TPC/SPC and a LDPC code over EPR4 channels (both have effective block size 4K, rate 0.94). The statistics are collected over 200 000 and 160 000 blocks of 4k bits, respectively. Although we cannot reliably predict the performance of the outer RS code at very low BER's such as 10^{-15} based on the limited observation, the results clearly indicate that TPC/SPC codes possess better burst error statistics than LDPC codes.

V. Conclusion

This paper investigates the potential of applying single-parity check turbo product codes to magnetic recording systems. A random interleaver, precoder and turbo equalizer are used to form a power structure of serial concatenation over PR channels. To maximize interleaving gain, multiple TPC/SPC codewords are combined before interleaving. Gains of 4.5–5 dB are observed over uncoded systems, revealing the performance of TPC/SPC codes comparable to that of LDPC codes. Block error statistics are examined and TPC/SPC codes are seen to have much smaller error bursts than LDPC codes. The results have shown that TPC/SPC codes should be seriously considered for potential application in future magnetic recording systems.

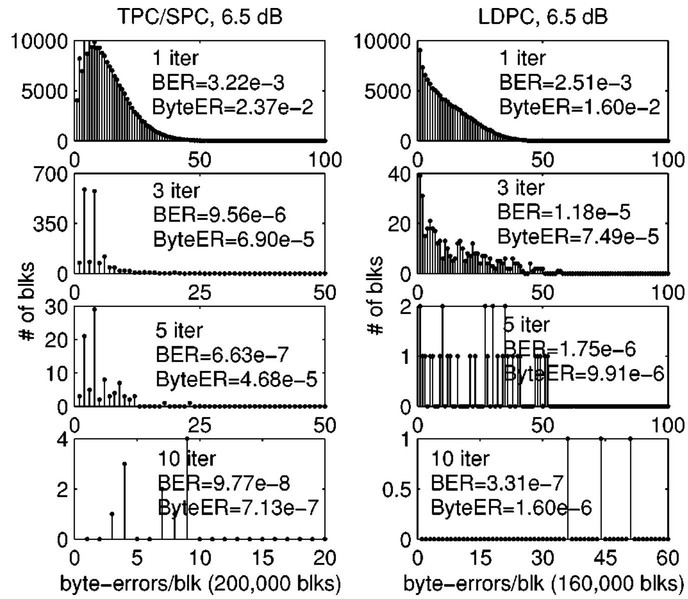

Fig. 4. Block error statistics (y-axis: number of byte errors in a block, x-axis: occurrence of such blocks, BER: bit error rate, ByteER: byte error rate).

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