A Class of $(\gamma\rho^{\gamma-1}, \rho^{\gamma}, \rho, \gamma, \{0, 1, \})$ Combinatorially Designed LDPC Codes with Application to ISI Channels

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I. Introduction

We investigate a systematic construction of regular low density parity check (LDPC) codes based on ($\gamma\rho^{\gamma-1}$, ρ^{γ}, ρ, γ,$\{0,1\}$) combinatorial designs. The proposed LDPC ensemble is (γ, ρ)-regular, linear-time encodable, and has rate $(1-\frac{1}{\rho})^{\gamma}$ and girth $\geq 2^{\gamma+1}$ (≥ 8). From one particular view point, the proposed codes are "loosened" finite geometry codes [2] where some lines are systematic removed. The loosened design rules offer at least two advantages: a lower decoding complexity and a richer code set.

The proposed codes are a subset of *Gallager's (γ, ρ)-regular random ensemble*, whose $m \times n$ parity check matrix H can be horizontally split into γ sub-matrices of dimensionality $\frac{m}{\gamma} \times n$ each, where each sub-matrix has uniform column weight 1 and row weight ρ. We show that the proposed codes contain a good combination of structure and (pseudo-)randomness, where the former enables low-cost implementation in hardware and the latter ensures reasonable performance by avoiding certain recurrent error patterns (eg. rectangular error patterns).

II. Code Construction

Consider $\gamma\rho^{\gamma-1}$ points and ρ^{γ} blocks, where blocks are labeled using γ-tuple (γ-dimensional) subscripts. Two blocks are said to be in the same plane if at least $(\gamma - 2)$ coefficients in the γ-tuple subscript are the same. For each direction along the γ dimensions, $\rho^{\gamma-2}$ parallel planes (call them "principal" planes) can be selected each containing ρ^2 blocks and collectively covering all blocks. The ρ^2 blocks in each principal plane can be evenly divided into ρ discrete "bundles" according to a predefined "bundle-rule" (will be discussed later), which results in altogether $\gamma\rho^{\gamma-1}$ bundles in $\gamma\rho^{\gamma-2}$ principle planes where no two bundles contain a same pair of blocks. Each bundle thus uniquely determines a point in that a point is incident only with blocks in the same bundle. Hence, there are altogether $\gamma\rho^{\gamma-1}$ well-defined points and ρ^{γ} well-defined blocks, such that each point is incident with γ blocks, each block with ρ points and no two blocks overlap on more than one point. This results in a $(\gamma\rho^{\gamma-1}, \rho^{\gamma}, \rho, \gamma, \{0, 1\})$-design which uniquely defines a (γ, ρ)-regular LDPC ensemble. The choice of bundle-rule directly affects the complexity as well as the performance of the resulting code. In addition to cyclic permutation (such as those used in finite geometry LDPC codes), other admissible choices include permutation tables constructed from M-sequences (using a cyclic shift register), congruential sequences and the arithmetics from the Galois field. For example, when $\rho = p^t$ where p is a prime number and t is any positive integer, the Zech logarithm in GF(p^t) can be used to define the bundle-rule.

[1] This work was supported by a Research Grant from Seagate Technology.

III. A Simple Example of $\gamma = 2$

To illustrate the properties and performance of the proposed LDPC codes, we study a simple case of $\gamma = 2$, which leads to a class of codes that are high-rate, systematic, quasi-cyclic, linear-time encodable and decodable, and free of length-4 and length-6 cycles. While the proposed codes are in general isomorphic with turbo product codes (TPC), this simplest case results in an equivalence to a 2-dimensional single parity check turbo product codes (TPC/SPC). The simplicity of this code makes it possible to compute the exact distance spectrum. We compare the output weight enumerators (OWE) of this code with the average OWE of the *Gallager random ensemble*. For the same parameters of code length and column weight, we see that the proposed structured code is actually better than the *Gallager random codes* with fewer codewords at low weights. This suggests that the performance of the proposed codes is above the average when an optimal decoder is available.

It should be pointed out that $(2, \rho)$-regular LDPC codes are not "good" codes in that they do not have an iterative threshold on AWGN channels. However, when $(2, \rho)$ codes are used with a modulator or channel that has memory, the modulator/channel will provide another level of parity check to the coded bits of the original LDPC code. Hence, when iterative decoding and demodulation/equalization is performed (as is the trend of most high-performance applications), $(2, \rho)$ codes are turned into "good" codes. Further, recent work has shown that regular LDPC codes are asymptotically optimal (i.e. reaching to the i.i.d. capacity of the channel) on a dicode channel [3]. We conjecture the result to be valid on general intersymbol interference (ISI) channels too.

One possible application for the proposed structured LDPC codes, especially the $(2, \rho)$ codes which are both simple and high-rate, is the digital recording systems. We evaluate the performance of a rate 0.88 and 0.94 code on PR4, EPR4, and E^2PR4 channels, respectively. Simulations show that with proper precoding, the proposed codes can perform as well as or slightly better than the random codes. Unlike random codes, the proposed structured LDPC codes can lend themselves to a low-complexity implementation for high-speed applications.

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0-7803-7728-1/03/$17.00 ©2003 IEEE.