Progressive Network Coding for Message-Forwarding in Ad-Hoc Wireless Networks

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***Abstract*— We consider multi-hop transmission from the source to the destination in ad-hoc wireless networks. Cooperative forwarding approaches in the framework of *progressive network coding* are proposed which generalize *coded cooperation* in a multi-hop context. In this framework, the source node and each succeeding relay node progressively decode what they receive from the previous stages and re-encode the messages to different parts of the parity bits from a single (network) codeword hop by hop. The maximal achievable rates for the multi-hop wireless networks using traditional *repetition-forward* and *progressive network coding* are analyzed with respect to different transmit power constraint and packet length allocation. The optimal number of relays are derived in each scheme. It is found that *progressive network coding* with adaptive packet length significantly increases the system throughput and improves the energy efficiency.**

I. Introduction

The signal relaying problem can be traced back to the work of van der Meulen [1]. Groundbreaking work done by Cover and El Gamal in [2] proposed several relaying strategies and extensively investigated information theoretic properties based on additive white Gaussian noise (AWGN) channels. Motivated by the flourishing wireless network, recent researches have largely focused on fading channels [3]-[8].

Ad-hoc networks, a concept different from and complimentary to fractured networks, can improve the system performance significantly by relaying signals in multiple hops between the source and the destination without any central control. As a cost to the flexibility of ad-hoc networks, ad-hoc networks suffer from the complexity of the routing problem, which is a critical issue to the system performance. Recently, a large number of papers appeared in the literature addressing the routing problem in ad-hoc networks (see, for example, [11][12] and the references therein). However, most of the routing strategies proposed are based on the traditional store-and-forward mechanism, or, the *repetition-forward* framework, in the jargon of *user cooperation*.

From the coding perspective, however, repeating the message bit-on-bit is very inefficient, since repetition codes are the weakest type of practical codes. A variety of insightful network examples have been constructed, first in the context of lossless networks and later extended to lossy and wireless networks, which demonstrate that (i) the traditional routing strategy does not come close to achieving the communication capacity in network settings, and (ii) *network coding*, generalization of repetition-forwarding by allowing intermediate relaying nodes to perform intelligent packet combining in the symbol level, is crucial in delivering end-to-end optimal network performance networks. Hence, networking coding can also be considered as generalization of routing.

This paper considers the multi-hop transmission problem in wireless ad-hoc networks. In a three-node scenario that consists of a source, a destination and a single relay, it was shown in [6] under the term of *coded cooperation* that, letting the source and the relay collaboratively transmit different parts of a single codeword can increase the system capacity and reduces the outage probability. Motivated by the encouraging coding gain enabled by *coded cooperation*, we propose here a generalized framework in the context of multi-hop transmission. The new framework, exploiting the technology of *progressive network coding*, allow each relay node in the ad-hoc networks to gather all they hear from previous hops, decode segments altogether, and re-encode and transmit a different (sub-)codeword. Hence, the message propagates along the route from the source to the destination in a wave-like fashion: as the wave front proceeds from one hop to another, part of it fades, i.e. the reception of a packet wirelessly originated from an early relay becomes increasingly weak or close to undetectable further down the stream; at the same time, the message wave is also re-strengthened at each hop, since the new relay re-generates and airs a new (sub) codeword pertaining to the message. The destination will, conceptually, receive all the (sub) codewords from the original source as well as each and every participating relay along the route, however weak some signals may be. It can therefore launch an iterative decoding by treating the (sub) codewords as outputs from the parallel branches of a giant *parallelly concatenated (network) code.* In practice, depending on the length of the routes and the severity of signal attenuation, it may suffices for the destination to include only the last few (sub) codewords in its iterative decoding.

This framework subsumes *coded cooperation* (when there is only one relay) as a simplified case, and can be considered as generalization of *coded cooperation* from the three-node scenario to multiple-node scenario. Equipped with the idea of *progressive network coding*, the traditional least-hop routing scheme may no longer be optimal. Then what is the appropriate (routing) topology in the new scenario? To answer this question, we model the system in the context of degraded broadcast channels (block Rayleigh fading channels are degraded broadcast channels), and analyze the maximal achievable rates for different forwarding schemes under power control and with packet length allocation. In addition, practical coding simulations are performed to verify the excellent performance of the proposed new scheme.

The rest of this paper is organized as follows. Section II introduces the system model used in this paper. Section III gives a quick review of the traditional routing and forwarding scheme in ad-hoc networks, and section IV describes the basic scheme of the *progressive network coding*. To find the optimal routing strategy for the traditional *repetition-forward* and *progressive network coding*, section V-C analyzes the maximal achievable rates for different schemes under power control and packet length allocation. Section VI verifies the excellent performance of *progressive network coding* by practical simulations, and section VII concludes the whole paper.

II. System Model

We study the problem of one source transmitting data to a destination in wireless ad-hoc networks. The destination is with a overall distance of d segmented by n hops from the source. In this paper, we consider a simple but generalizable model with the ad-hoc network forms a line network, with the source on one end, the destination on the other, and all other nodes uniformly and randomly distributed in a straight line between them. For ease of analysis, we further assume that the network is sufficiently dense, so that we can find a relay wherever we want it to be.

We note that the model of line network, although simple, is not unrealistic: after a route is found from the source to the destination, the relay nodes along the route essentially form a line network. The one rigid constraint we have imposed here is that they form a straight line. This is not inherent to our analysis and our method generalizes to more practical scenarios. The straight-line geometry is assumed so that we can quickly and easily compute the distance between any two nodes, evaluate the corresponding signal attenuation, and hence launch an accurate and quantitative analysis of the achievable rates for different routing strategies.

The general form of the signal received over a channel at time t is given by:

$$y(t) = \sqrt{E_s h}\, x(t) + Z(t), \tag{1}$$

where E_s is the signal energy, h is the path-loss, and $Z(t)$ is the additive white Gaussian noise (AWGN) with power spectrum density N_0. The path-loss h follows an exponentially attenuating law as:

$$h = l^{-\delta}, \tag{2}$$

where l is the distance between the transmitter and the receiver, and δ is the attenuation factor, which is an integer between 2 and 5. Additionally, we assume all the terminals have the same average power constraint P; they thus have the same transmit signal-to-noise ratio (SNR) γ_0, which is defined as

$$\gamma_0 \triangleq P/N_0. \tag{3}$$

The SNR γ_k is defined as the received SNR of two nodes separated by k hops. Since all the relays are evenly distributed in one line. We can write

$$\gamma_k \triangleq \gamma_0 \left(\frac{kd}{n}\right)^{-\delta}. \tag{4}$$

Furthermore, for simplicity, we assume the system operates in the half-duplex mode, and the relays transmits one by one after hearing the previous relay finish transmission. Additionally, from the source to the destination, the nodes are indexed from 1 to $n+1$.

III. The background knowledge for the routing problem in ad-hoc wireless networks

We start by discussing the routing problem in a simple scenario as shown in Fig. 1, where node 2 helps node 1 (the source) transmit data to the destination D. In the traditional *repetition forward* strategy, node 1 first transmits its data packet, and node 2 decodes and regenerates terminal 1's packet and forwards it to the destination.

Since the path-loss increases with the distance, node 1 may not reach the destination directly. One conventional method is to consider that each node has a "coverage radius", and try to use the least number of hops possible to reach the destination.

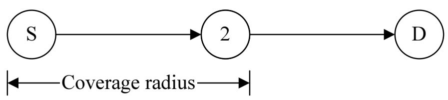

Fig. 1. Traditional *repetition forward* of 2 hops. The routing strategy is to use the least hops as possible.

The least-hop routing strategy has a low complexity, but the transmission rate may be suboptimal, the transmission reliability is poor, and the system is vulnerable to link failure.

Realistic wireless networks may operate in a dense scenario, where each terminal may have multiple nodes to serve as the potential relays. In that case, it might pays to allow more than one relay in a node's coverage radius to forward messages.

Consider a slightly denser network in Fig.2, where 4 terminals transmit data to the destination, and each terminal has 2 other downstream terminals within its coverage radius. We call this routing strategy "redundant repetition".

For repetition-based forwarding, the transmit packet for each node is shown in Fig.3. We see that each node has the same packet length N, and transmits the same code codeword with systematic bits S_1 and parity check bits P_1.

From the coding point of view, *redundant-repetition* exploit only the trivial repetition code, thus getting poor coding gain. Since *redundant-repetition* uses extra bandwidth compared to least-hop relaying, the small gain it achieves (via repetition codes) may not over-weigh the cost for power and bandwidth.

IV. Progressive network coding

Coded cooperation[6] is an efficient technique for user cooperation, where the source and the relay transmit different parts of a single codeword. When extending the *coded cooperation* to the multi-hop networks, this becomes *progressive network coding*. In the new context, each node transmits a different part of a single network codeword, and each node combines all the signals received from the previous nodes to extract the information.

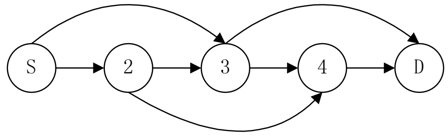

Fig. 2. Traditional repetition-and-forward when there are four additional terminals, two relays in each node's coverage.

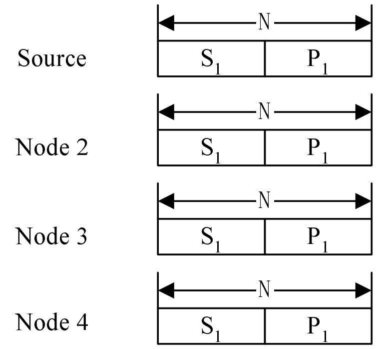

Fig. 3. Transmit packets in the traditional storage-and-forward routing when there are two nodes in coverage radius.

Consider the same scenario as *redundant-repetition* (Fig.2). Now, instead of re-generating and repeating what it hears, each relay re-encodes the message using a nontrivial code. In the more general context, we can model the system as one that all the nodes in the downstream hears a node, although some of the signals may be so weak as to close to useless. The data flow of *progressive network coding* can then be illustrated in Fig. 4, where each node can potentially combine all the previous packets to deduce the data conveyed by the source. Hence, this system progressively encodes all the packets into one single codeword, or, put another way, with with additional hop, the network code is strengthened by including new parity bits and more checks.

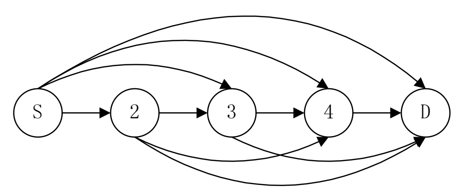

Fig. 4. The data follow of Progressive network coding

To clarify the spirit of progressive network coding, we show the transmit packets for each terminal in Fig. 5. Comparing between Fig. 5 and Fig. 3, we see that unlike *repetition-forward*, in *progressive network coding*, each node will transmit different sets of parity check bits of possibly different sizes.

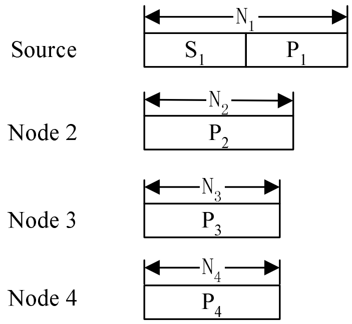

Fig. 5. The transmit packet of each terminal for progressive network coding. N_1,N_2,N_3 and N_4 are the packet length for each node.

V. Performance analysis

To provide a quantitative evaluation of the performance and to find the optimal routing strategy, we perform theoretical analysis for traditional *repetition-forward* and *progressive network coding*. We first consider the case when the transmit power and packet lengths are uniformly allocated among all the nodes, and analyze the maximal achievable rates for the two forwarding schemes. Next, we consider more general cases where power and packet lengths are varying and adjustable between different nodes.

A. Fix transmit power and packet length

1) Repetition forward: For the repetition-based message forwarding, since the transmission breaks whenever any node fails to extract the data from the combined signals of all its precessors, the maximal achievable rate is the minimum of all the achievable rates from the source to any of the nodes down the stream. Mathematically, we can write:

$$C_{rep} = \min(R_{1,2}^{rep}, R_{1,3}^{rep}, R_{1,4}^{rep}, \cdots, R_{1,n}^{rep}, R_{1,n+1}^{rep}), \quad (5)$$

where $R_{1,k}^{rep}$ is the achievable rate between the source (i.e. node 1) and node k. Since the achievable rate for repetition transmission can be calculated using the classic Shannon formula whose equivalent SNR is the sum of the SNRs of all the previous channels, $R_{1,k}^{rep}$ satisfies

$$R_{1,k}^{rep} = \frac{1}{n}\log_2\left(1+\sum_{i=1}^{k-1}\gamma_i\right). \quad (6)$$

From (6), we see that $R_{1,2}^{rep} = \frac{1}{n}\log_2(1+\gamma_1)$ is the minimum among $R_{1,k}^{rep}$. Combining (5) and (6), we find that the maximum achievable rate from the source to the destination is constrained by the first hop (recall that the nodes are separated with equal distances)

$$\begin{aligned} C_{rep} =& R_{1,2}^{rep}, \\ =& \frac{1}{n}\log_2(1+\gamma_1), \\ =& \frac{1}{n}\log_2\left(1+\gamma_0\left(\frac{d}{n}\right)^{-\delta}\right). \end{aligned} \quad (7)$$

To obtain the optimum number of hops to achieve the maximal rate (i.e. how many relays to use), we solve the following equation

$$\frac{\partial \log_2\left(1+\gamma_0\left(\frac{d}{n}\right)^{-\delta}\right)/n}{\partial n} = 0, \quad (8)$$

and get the following solution

$$n^* = \left\lfloor \left(\frac{1-\delta-LambertW((1-\delta)e^{1-\delta})}{LambertW((1-\delta)e^{1-\delta})\gamma_0 d^{-\delta}}\right)^{\frac{1}{\delta-1}} \right\rfloor, \quad (9)$$

where function $LambertW(.)$ satisfies

$$LambertW(x)\exp(LambertW(x)) = x. \quad (10)$$

Therefore, the optimal number of hops n^* for maximizing the data rate can be uniquely determined from given parameter of overall distance d and attenuation factor δ. In a typical scenario, substituting $d = 10$, and $\delta = 4$ in (9), we get the optimal number of hops n^* is 5.

Substitute (9) back into (7), we get the maximal rate of *repetition-forward*:

$$C_{rep}^* = C_{rep}(n^*). \quad (11)$$

Specifically, when n tends to infinity, we have

$$\begin{aligned} \lim_{n\to\infty} C_{rep}^*(n) =& \lim_{n\to\infty}\frac{1}{n}\log_2\left(1+\gamma_0\left(\frac{d}{n}\right)^{-\delta}\right), \\ =& \lim_{n\to\infty}\frac{\gamma_0\, d^{-\delta}n^{\delta-1}\delta}{(1+\gamma_0\, d^{-\delta}n^{\delta})\ln(2)}, \\ =& 0. \end{aligned} \quad (12)$$

From (12), we see that for traditional repetition based relaying, one cannot effectively increase the achievable rate of the system by simply adding more relays along the way, because the maximal achievable rate is asymptotically zero when the number of hops trends to infinity (and each hop trends to infinitesimal).

2) Progressive Network Coding: Similar to the repetition based forwarding, the transmission breaks if any of the intermediate node does not decode correctly, so the achievable rate can be written as the following:

$$C_{pnc} = \min(R^{pnc}_{1,2}, R^{pnc}_{1,3}, \cdots, R^{pnc}_{1,n}, R^{pnc}_{1,n+1}), \quad (13)$$

where $R^{pnc}_{1,i}$ is the maximal achievable rate between the source and node k, Now since each node transmits different parity bits, the achievable rates will be the sum of the Shannon formulas rather than the sum of the SNRs inside the Shannon formula, that is,

$$R^{pnc}_{1,i} = \frac{1}{n} \sum_{j=1}^{i-1} \Omega_{i-j}, \quad (14)$$

where Ω_k, the capacity of direct transmission between two nodes separated by k hops, is given by

$$\Omega_i = \log_2(1 + \gamma_i). \quad (15)$$

From (13) and (14), we can easily see that the first hope is again the bottle-neck. Hence,

$$C_{pnc} = R^{pnc}_{1,2} = \Omega_1/n = \frac{1}{n} \log_2(1 + \gamma_1). \quad (16)$$

Comparing (16) with (7), we find that *fixed progressive network coding* and *fixed repetition-forward* have the same maximal achievable rate if one fixes the transmit power and packet length for each node.

B. Transmit Power and Packet Length Allocation among Different Nodes

1) Repetition forward: For *repetition-forward*, we can only play with the transmit power because the packet lengths for all the nodes are equal.

By allowing dynamic power allocation among the nodes, the maximal achievable rate for repetition based forward can be written as following:

$$C_{rep} = \max_{\alpha_1,\alpha_2,\cdots,\alpha_n} \min(\tilde{R}^{rep}_{1,2}, \tilde{R}^{rep}_{1,3}, \cdots, \tilde{R}^{rep}_{1,n}, \tilde{R}^{rep}_{1,n+1}), \quad (17)$$

where $\tilde{R}_{1,k}$ is the maximal achievable rate of two nodes of k hops away.

We can further write:

$$\tilde{R}_{1,k} = \frac{1}{n} \log_2 \left(1 + \sum_{i=1}^{k-1} \alpha_i \gamma_i \right), \quad (18)$$

where α_i is the power allocation factor of the ith node, and satisfies

$$\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1. \quad (19)$$

The solution to (18) is achieved when

$$\tilde{R}^{rep}_{1,2} = \tilde{R}^{rep}_{1,3} = \cdots = \tilde{R}^{rep}_{1,n+1}. \quad (20)$$

Substituting (18) in (20), we can rewrite the problem in a compact matrix form:

$$\begin{bmatrix} \gamma_2 - \gamma_1 & \gamma_1 & & & \\ \gamma_3 - \gamma_1 & \gamma_2 & \gamma_1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ \gamma_n - \gamma_1 & \gamma_{n-1} & \gamma_{n-2} & \cdots & \gamma_1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (21)$$

After a tedious but otherwise simple algebra, we solve (21) and get the maximum achievable rate of *repetition-forward* with dynamic power allocation:

$$\tilde{C}_{rep} = \tilde{R}^{rep}_{1,2} = \frac{1}{n} \log_2(1 + n\alpha_1\gamma_1). \quad (22)$$

2) Progressive Network Coding: For *progressive network coding*, since the packet length for each node is adjustable, we allocate different packet lengths to different nodes while keeping their transmit power the same. We note the true optimal performance is achieved when both the transmit power and the packet lengths are optimally adjusted, but that appears to be analytically intractable. Hence, here, we consider only the adaptation of packet lengths and abuse the term "optimal".

Following the same line of discussion as in the power allocation for *repetition-forward*, we can write

$$\tilde{C}_{ncc} = \max_{K_1,K_2,\cdots,K_n} \min(\tilde{R}^{pnc}_{1,2}, \tilde{R}^{pnc}_{1,3}, \cdots, \tilde{R}^{pnc}_{1,n}, \tilde{R}^{pnc}_{1,n+1}), \quad (23)$$

where $\tilde{R}^{pnc}_{1,i}$, the maximal achievable rate between the source and node i with dynamic packet length allocation, follows

$$\tilde{R}^{pnc}_{1,i} = \sum_{j=1}^{i-1} K_j \Omega_{i-j}, \quad (24)$$

and, K_j, the packet length factor of the jth node, satisfies

$$K_1 + K_2 + \cdots + K_n = 1. \quad (25)$$

To solve (23), using a similar process as the power allocation in *repetition-forward*, the solution for (19) is achieved when

$$\tilde{R}^{pnc}_{1,2} = \tilde{R}^{pnc}_{1,3} = \cdots = \tilde{R}^{pnc}_{1,n+1}. \quad (26)$$

We combine (25) with (26) to form a compact matrix representation:

$$\begin{bmatrix} \Omega_2-\Omega_1 & \Omega_1 & & & \\ \Omega_3-\Omega_1 & \Omega_2 & \Omega_1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ \Omega_n-\Omega_1 & \Omega_{n-1} & \Omega_{n-2} & \cdots & \Omega_1 \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ \vdots \\ \vdots \\ K_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad (27)$$

whose solution leads to the maximal achievable rate for the *progressive network coding* as:

$$\tilde{C}_{pnc} = \tilde{R}^{pnc}_{1,2} = K_1\Omega_1. \quad (28)$$

C. Numerical results

To evaluate the performance of different forwarding schemes, we provide the numerical results when the distance between the source and the destination is $d = 10$, the transmit SNR is $\gamma_0 = 30$ dB, and the fading factor is $\delta = 4$.

Fig. 6 demonstrates the maximal achievable rate vs different numbers of hops when the transmit power and the packet length for each node are fixed. We first note the the maximal achieve rate of repetition-forward gets maximal value at 5, which complies with the analytical results from equation (9) in section .

In addition, although somewhat disappointing, we observe from this figure that *progressive network coding* does not exhibit any advantage in terms of the maximal achievable rate. Further, the achievable rate does not always increase with the number of hops, but is maximized when the the number of hops in use satisfies (9).

Fig. 7 and Fig. 8 show the optimal power allocation factors for *repetition-forward* and the optimal packet length factors for *progressive network coding*, respectively. From these two figures, it is interesting to observe that the first transmitting node (or the first-hop transmission) is the most important in ensuring a high information rate, and deserves either a larger transmit power or a longer packet length than the others. This is possibly because that the first transmitting node has the largest impact since it is potentially over-heard by all the succeeding nodes. A closer look into these figures reveals that the difference between the nodes' power allocation factors in *repetition-forward* is actually quite small, which suggests that equal power allocation is also close to optimal. On the other hand, the gap between the nodes' packet length factors in *progressive network coding* is significantly larger. For the case we studied, we find that the first node is expected to transmit for more than 4 times longer than the rest of the nodes.

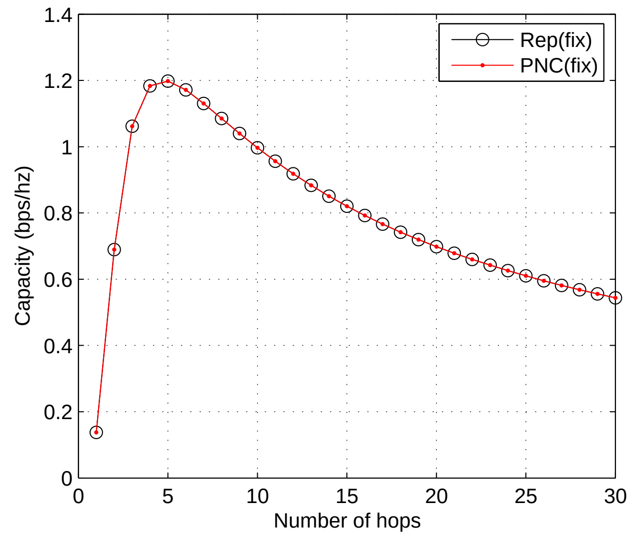

Fig. 6. The maximal achievable rate from the farthest node to the destination vs different number of relays. The transmit power and packet length are all equal. The distance is 10, fading factor $\delta = 4$, and the transmit SNR $\gamma_0 = 30$ dB.

Fig. 9 shows the maximal achievable rates for *repetition-forward* with dynamic power allocation and *progressive network coding* with dynamic packet length allocation. Two observations are notable here. First, as indicated from Fig. 7, adjusting transmit power improves the information rate for *repetition-forward* but only marginally. Second, optimal packet length allocation significantly improves the achievable rates for *progressive network coding*. The gain keeps increasing with the number of hops. The immediate implication is that one is better off taking all the nodes available in the system as potential relays when *progressive network coding* is used. This is drastically different from the case of *repetition-forward*, where the asymptotic information rate goes to zero as the number of hops increases without bound.

VI. Practical Network Coding Schemes and Experiment Results

A. Practical coding scheme

The discussion in the previous sections reveal the idea of *progressive network coding* and provides a theoretic support for its efficiency. In this section, we demonstrate the feasibility of *progressive network coding* and verify its superb performance through practical code construction and computer simulations.

Since the nature of *progressive network coding* requires the code rate to be flexible, or more precisely,

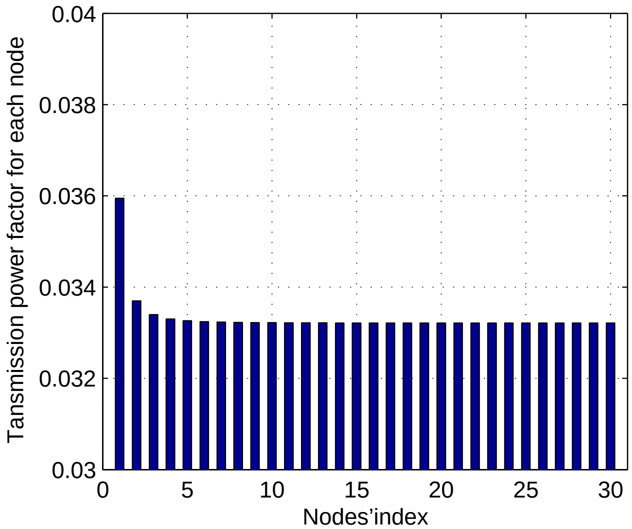

Fig. 7. Transmit power factor for each node with 30 hops. The distance is 10, fading factor is 4, and the transmit SNR is 30 dB.

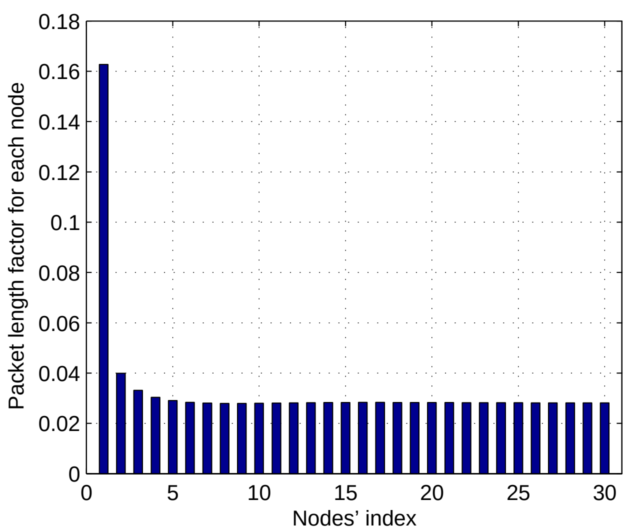

Fig. 8. Transmit time factor for each node with 30 hops. The distance is 10, fading factor is 4, and the transmit SNR is 30 dB.

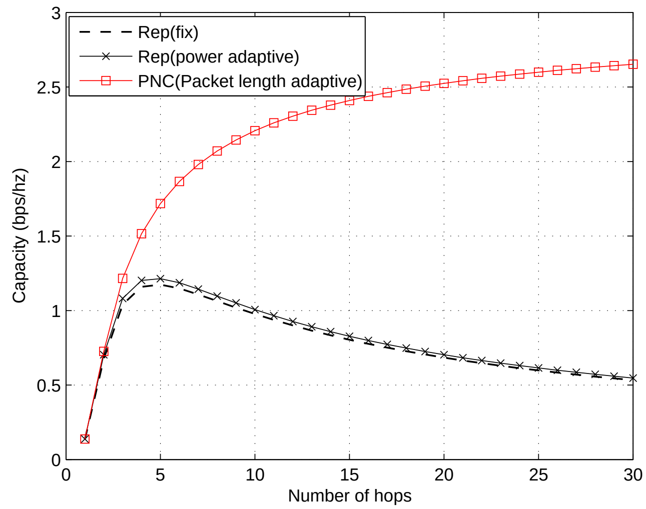

Fig. 9. The maximal achievable rate from the farthest node to the destination when *repetition-forward* takes on the optimal power allocation and *progressive network coding* takes on the optimal packet length allocation. The distance is 10, fading factor $\delta = 4$, and the transmit SNR $\gamma_0 = 30$ dB. For comparison purpose, we also plotted the achievable rate with even power and packet length allocation.

it calls for a family of rate-compatible or structurally-embedded codes. There are a large number of candidate codes that can be tailored to serve our needs, including, for example, rate-compatible punctured convolutional (RCPC) codes, rate-compatible punctured turbo (RCTC) codes, and the rateless Luby-Transform codes [13] and Raptor codes [14].

In this paper, we propose to use a distributed lower-triangular low-density parity-check (LT-LDPC) codes [8], whose parity-check matrix is illustrated in Fig. 10.

LT-LDPC codes are a special class of linear-time encodable LDPC codes. The parity check matrix of a systematic LT-LDPC code consists of two parts, a sparse (and random) matrix P on the left and a sparse lower triangular matrix Q with all ones in the main diagonal on the right, i.e. $H = [P, Q]$. Adapting this code in the *progressive network coding* framework, the H matrix will be decomposed to sub matrices, such that each node uses the sub matrix corresponding to the previous node to decode the data and use the submatrix corresponding to itself to encode and generate a new set of parity bits.

B. simulations

In our simulation, we assume that the source reaches the destination in 4 hops with the distance of each hop being 1. That is, the distance from the source to the destination is 4, We assume that the fading factor $\delta = 4$, such that the received SNR for two nodes with k hops apart, γ_k, satisfies

$$\begin{aligned} \gamma_1 =& \gamma_0, \\ \gamma_2 =& \frac{1}{16}\gamma_0, \\ \gamma_3 =& \frac{1}{81}\gamma_0, \\ \gamma_4 =& \frac{1}{256}\gamma_0. \end{aligned} \tag{29}$$

Let the block size of the source's information data be 1000 and the base code rate be $1/3$. Hence, for

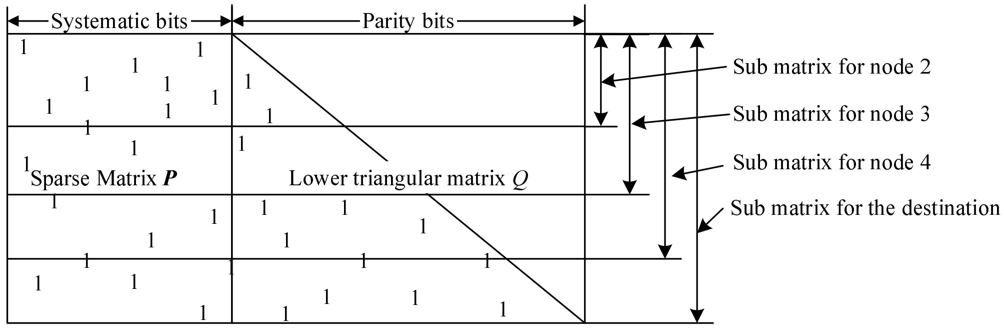

Fig. 10. The parity-check matrix for the distributed LT-LDPC codes. Four transmission hops in progressive network coding.

repetition-forward, the code length for each node is 3000. For *progressive network coding*, we use the packet length factors of 0.46, 0.18, 0.18 and 0.18 for the source, node 2, node 3, node 4 respectively (these values are obtained from the solution to (27)). Therefore, the packets transmitted from the source, node 2, node 3, node 4 have lengths of 1380 (1000 systematic bits and 380 parities), 540 (parity only), 540 (parity only), and 540 (parity only), respectively. With this setup, *progressive network coding* uses only $1/4$ of the total bandwidth of *repetition-forward*. To decode the signals, each node uses the standard sum-product message-passing algorithm with a maximum of 30 iterations to extract the systematic bits from all of the previous nodes before re-encoding them.

The simulation results are shown in Fig. 11. We see that *repetition-forward* achieves a bit error rate (BER) of 10^{-6} at about 13 dB, whereas *progressive network coding* method needs only about 7 dB. Hence, *progressive network coding* outperforms traditional repetition-forward by some 6 dB, and this gain is achieved in addition to 75% of bandwidth saving!

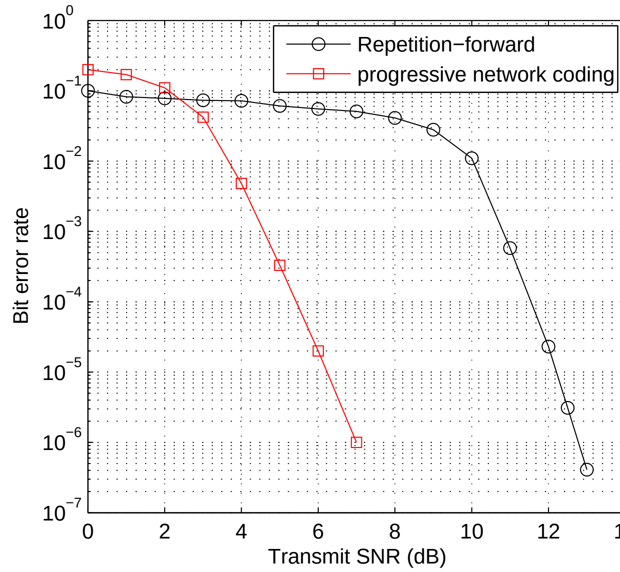

Fig. 11. Comparison between storage-and-forward and progressive network coding in Gaussian channels

VII. Conclusion

In ad-hoc wireless networks, data packets may be relayed in multiple hops before reaching the destination. The efficiency of the routing or forwarding strategy is an important factor to the system performance.

Since the traditional *repetition-forward* approach is suboptimal or, in some cases, even bandwidth-wasteful, we propose to exploit network coding to improve the routing efficiency. The proposed *progressive network coding* framework allows each node along the path to progressively decode and re-encode the message, and to forward different sets of parity bits. The maximal achievable rates for *repetition-forward* and *progressive network coding* under dynamic transmit power and packet length allocation are analyzed, and the optimal number of hops are derived for each case. Theoretical analysis shows that *progressive network coding* can potentially make good use of all the nodes in the system to continually improve the system performance, whereas attempting to use too many relays leads to a vanishing information rate for *repetition-forward*. In addition to theoretic analysis, practical coding solutions using lower-triangular LDPC codes are demonstrated and simulation confirms the excellent performance of *progressive network coding* by demonstrating some 6 dB gain over traditional *repetition-forward*.

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